No. of Printed Pages : 3 **MMT-002**

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00182

Term-End Examination

December, 2014

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Question no.* ***5*** *is* ***compulsory****. Answer any* ***three*** *questions from questions no.* ***1*** *to* ***4****. Use of calculators is* ***not*** *allowed.*

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1. (a) Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be a linear transformation defined by

$$T(x, y, z) = (3x + 2y - 4z,\ x - 5y + 3z).$$

Find the matrix of T relative to the bases $\{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ of $\mathbf{R}^3$ and $\{(1, 3), (2, 5)\}$ of $\mathbf{R}^2$. *3*

(b) Find the spectral decomposition of

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$ *2*

2. (a) Find a QR-decomposition of A, and hence find a least-squares solution of the system $Ax = b$, where 3

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}.$$

(b) Write all possible Jordan canonical forms of a 5×5 matrix having

$$(t-2)^2 (t-3)(t-4)$$

as minimal polynomial. 2

3. Find the singular value decomposition of

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}.$$ 5

4. (a) Prove that a non-zero nilpotent operator is not diagonalisable. 2

(b) Check whether $A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}$ is unitarily diagonalisable. If it is, find a unitary matrix U such that $U^* A U$ is diagonal. Otherwise, obtain the Schur decomposition of A. 3

5. Which of the following statements is true ? Give reasons for your answers. *10*

(a) The sum of two unitarily diagonalisable matrices is unitarily diagonalisable.

(b) An invertible matrix must be positive definite.

(c) A and A^tA have the same rank, for any matrix A.

(d) If A is a diagonalisable matrix, the geometric multiplicity of each of its eigenvalues is 1.

(e) If N is a nilpotent matrix, then e^N is also nilpotent.
