B.Tech. – VIEP – COMPUTER SCIENCE AND ENGINEERING (BTCSVI)

00115

Term-End Examination

December, 2014

BICS-008 : DISCRETE MATHEMATICAL STRUCTURES

Time : 3 hours *Maximum Marks : 70*

Note : *Part A is* ***compulsory*** *and carries 14 marks. Answer any* ***four*** *questions from Part B which carries 56 marks. Answer all parts of a question at one place.*

PART A

1. Give an example of a relation which is symmetric, transitive but not reflexive on {a, b, c}. *2*

2. Let f(x) = x + 2, g(x) = x – 2 and h(x) = 3x for $x \in R$, where R is the set of real numbers. Find

(i) $g \circ f$

(ii) $h \circ g$ *2*

3. In a lattice show that *2*

$$(a \wedge b) \vee (c \vee d) \leq (a \vee c) \wedge (b \vee d).$$

4. Let $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$. Find whether f is an even or odd permutation. 2

5. If $|P \cup Q \cup R| = |P| + |Q| + |R|$, then which of the following is true ? 2

(i) $(P \cap Q) = \phi$

(ii) $(R \cap Q) \cup (P \cap R) = \phi$

(iii) $(P \cap Q \cap R) = \phi$

6. Construct the truth table to determine whether the statement $(P \rightarrow (P \rightarrow Q \wedge \sim Q))$ is a tautology, absurdity or contingency. 2

7. Find the DNF of the expression

$E(x, y, z) = (x + y + z)(x + y + z')(x' + y + z)$. 2

PART B

*Attempt any **four** questions.*

8. (a) Let Z^+ denote the set of positive integers and Z denote the set of integers. Let $f : Z^+ \rightarrow Z$ be defined by

$$f(n) = \begin{cases} n/2, & \text{if } n \text{ is even} \\ (1-n)/2, & \text{if } n \text{ is odd} \end{cases}$$

Prove that f is a bijection and find f^{-1}. 7

(b) Let P = { {1, 2}, {3, 4}, {5} } be a partition of the set S = {1, 2, 3, 4, 5}. Construct an equivalence relation R on S so that the equivalence classes with respect to R are precisely the members of P. 4

(c) It was found that in first year of computer engineering, out of 80 students 50 knew 'C' language, 55 knew 'Basic' and 25 knew 'C++, while 8 did not know any language.

How many knew all the three languages ? 3

9. (a) Let G = {EVEN, ODD} and binary relation $\oplus$ is defined as

$\oplus$	EVEN	ODD
EVEN	EVEN	ODD
ODD	ODD	EVEN

Show that (G, $\oplus$) is a group. 7

(b) Prove with rule of Inference or disprove :

If it is hot today or raining today then it is no fun to snow ski today.

It is no fun to snow ski today.

Therefore, it is hot today.

UNIVERSE = DAYS 7

10. (a) Let A be a given finite set and p(A) its power set. Let $\subseteq$ be the inclusion relation on the elements of p(A). Draw Hasse diagram of (p(A), $\subseteq$) for

(i) $A = \{a\}$

(ii) $A = \{a, b\}$

(iii) $A = \{a, b, c, d\}$ 7

(b) Simplify the Boolean function

$$f(x, y, z) = x'y'z + x'yz' + xyz'$$

using K-map and draw the circuit diagram for it. 7

11. (a) Show that

$(((P \vee \sim Q) \rightarrow R \leftrightarrow S) \vee \sim ((P \vee \sim Q) \rightarrow R) \leftrightarrow S)$

is a tautology. 7

(b) Without constructing the truth table, verify if the following statements are logically equivalent :

$$(P \leftrightarrow Q) \equiv (\sim P \vee Q) \vee (P \wedge Q)$$ 7

12. (a) Solve the Recurrence relation,

$S(k) - 7\,S(k-1) + 10\,S(k-2) = 6 + 8k$ with $S(0) = 1$ and $S(1) = 2$. 7

(b) Differentiate between the following and give examples of each : 7

(i) Walk and Path

(ii) Euler's Graph and Hamiltonian Graph