No. of Printed Pages : 6

ET-101(A)

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) / BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI

Term-End Examination

December, 2014

01265

ET-101 (A) : MATHEMATICS – I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of calculator is allowed.*

1. Answer any ***five*** of the following : *5×4=20*

(a) Evaluate any ***one*** of the following limits :

(i) $\lim_{x \to 1} \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$

(ii) $\lim_{x \to 0} \left[\frac{1}{x} - \frac{1}{x^2} \log (1 + x)\right]$

(b) For what value of 'k' is the following function continuous at x = 1

$$f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & , \quad \text{for } x \neq 1 \\ k & , \quad \text{for } x = 1 \end{cases}$$

(c) If $y = e^{3 \log x + 2x}$, prove that

$$\frac{dy}{dx} = x^2 (2x + 3) e^{2x}.$$

(d) Find the equations of the tangent and the normal to the curve

$$x^3 + x^2y - y^3 + 7 = 0,$$

at the point $x = 2, y = 3$.

(e) Find $\frac{dy}{dx}$, where $y = \text{cosec}^{-1}\left[\frac{1}{2x\sqrt{1 - x^2}}\right]$.

(f) If $u = x^2 - 2y$, $v = x + y + z$, $w = x - 2y + 3z$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

2. Answer any ***four*** of the following : *4×4=16*

(a) Evaluate any ***two*** of the following :

(i) $\int \sec^4 x \tan x \, dx$

(ii) $\int_1^2 (e^{3x} + 3x^2) \, dx$

(iii) $\int \frac{\sqrt{x}}{1 + 4\sqrt{x}} \, dx$

(b) Evaluate any ***one*** of the following :

(i) $\int_1^2 x^2 \log x \ dx$

(ii) $\int_0^3 \sqrt{x+1} \ dx$

(c) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.

(d) Solve any ***one*** of the following :

(i) $\frac{dy}{dx} = e^{2x-3y} + 4x^2 e^{-3y}$

(ii) $\frac{x\,dx + y\,dy}{x^2 + y^2} + dy = 0$

(iii) $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$

(e) Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ using Simpson's $\frac{1}{3}$ rule taking $h = \frac{1}{4}$. Hence compute an approximate value of π.

3. Answer any ***four*** of the following : *4×4=16*

(a) (i) If $\mathbf{a} = 4\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{b} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$, find the projection of **b** on **a**.

(ii) Find a unit vector perpendicular to the vectors $\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$.

(b) Using the line integral, compute the work done by the forces

$$\mathbf{F} = (2y + 3)\,\hat{\mathbf{i}} + xz\,\hat{\mathbf{j}} + (yz - x)\,\hat{\mathbf{k}},$$

when it moves a particle from the point (0, 0, 0) to the point (2, 1, 1) along the curve $x = 2t^2$, $y = t$, $z = t^3$.

(c) A particle acted on by constant forces $4\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, is displaced from the point $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ to the point $5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + \hat{\mathbf{k}}$. Find the work done by the forces.

(d) If $\mathbf{F} = (x + y + 1)\,\hat{\mathbf{i}} + \hat{\mathbf{j}} - (x + y)\,\hat{\mathbf{k}}$, show that

$$\mathbf{F} \cdot \text{curl } \mathbf{F} = 0.$$

(e) If $\mathbf{F} = (x + y + az)\,\hat{\mathbf{i}} + (bx + 2y - z)\,\hat{\mathbf{j}} + (x + cy + 2z)\,\hat{\mathbf{k}}$,

find a, b, c such that curl $\mathbf{F} = 0$.

Then find ϕ such that $\mathbf{F} = \nabla\phi$.

4. Answer any ***six*** of the following : $6 \times 3 = 18$

(a) Prove that

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

(b) Under what conditions is the rank of the following matrix 3 ?

$$A = \begin{bmatrix} 2 & 4 & 2 \\ 2 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$$

(c) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ -3 & -6 & -9 \end{bmatrix}$$

(d) Solve the following equations by matrix method :

$$x - 2y + 3z = 4$$
$$2x + y - 3z = 5$$
$$-x + y + 2z = 3$$

(e) Solve

$$\begin{vmatrix} x+p & q & r \\ q & x+r & p \\ r & p & x+q \end{vmatrix} = 0$$

(f) For what values of x, is the matrix

$$\begin{bmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{bmatrix}$$

singular ?

(g) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 3 \end{bmatrix}$$

(h) Show that

$$\begin{bmatrix} 3 & 9-6i & -2+7i \\ 9+6i & -4 & 3+2i \\ -2-7i & 3-2i & 6 \end{bmatrix}$$

is a Hermitian matrix.