MCA (Revised)

## Term-End Examination <br> December, 2014

# MCS-033 : ADVANCED DISCRETE MATHEMATICS 

Time : 2 hours
Maximum Marks : 50
Note: Question number 1 is compulsory. Attempt any three questions from the rest.

1. (a) Differentiate methods of 'inspection' and 'telescoping sums' on relevant attributes. Cite an example application for each.
(b) Explain the Tower of Hanoi problem with an example.
(c) Suppose that the sequence $\left\{\mathrm{a}_{\mathrm{n}}\right\} \mathrm{n} \geq 0$, has the generating function $\mathrm{A}(\mathrm{z})$. Then, the generating function $B(z)$ for the sequence $\left\{b_{n}\right\} n \geq 0$, where $b_{n}=a_{n}-a_{n-1}$ for $n \geq 1$, and $b_{0}=a_{0}$, is given by $\frac{B(z)}{A(z)}=(1-z)$.
(d) State and prove Handshaking theorem. 4
(e) Find the chromatic number of the following graph :

2. (a) Show that $\mathrm{K}_{5}$ is not planar.
(b) Which of the following graphs are trees, and why?

(a)

(c)

(b)

(d)
3. (a) Show that for a sub-graph H of a Graph G, $\Delta \mathrm{H} \leq \Delta \mathrm{G}$.
(b) Solve the recurrence relation

$$
\begin{align*}
& u_{n}=2 u_{n-1}+2^{n}-1 \text { where } n \geq 1 \text { and } \\
& u_{0}=0 \tag{5}
\end{align*}
$$

4. (a) Evaluate the sum $\sum_{k=1} k 3^{k} C(n, k)$, using

> generating function technique.
(b) Prove that $a_{n}=\frac{3 n}{2}-2$ is a solution of the recurrence $a_{n}=2 a_{(n / 2)}+2$, where $n$ is a power of 2 and $a_{2}=1$.
5. (a) Solve the third order recurrence $u_{n}-9 u_{n-1}+26 u_{n-2}-24 u_{n-3}=0$, where $n \geq 3$ with the initial conditions $\mathrm{u}_{0}=6, \mathrm{u}_{1}=17$ and $\mathrm{u}_{2}=53$.
(b) Anil takes a loan of $r$ rupees which is to be paid back in $t$ months. If $i$ is the interest rate per month for the loan, what constant payment $p$ must he make at the end of each period?

