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MIN-007

M.S. IN SOFTWARE TECHNOLOGIES (MSST)

00259

Term-End Examination

December, 2014

MIN-007 : MATHEMATICAL FOUNDATION OF COMPUTER SCIENCE

Time : 3 hours *Maximum Marks : 100*

Note :

(i) *Section I is* ***compulsory****.*

(ii) *In Section II, solve any* ***five*** *questions.*

(iii) *Assume suitable data wherever required.*

(iv) *Draw suitable sketches wherever required.*

SECTION I

1. (a) Note : $Z_n = \{0, 1, 2, \ldots n-1\}$, for any operation $*$, $a*_n b = (a*b) \bmod n$;

Show that, the algebraic system $< Z_6, +_6, *_6 >$ is a field, where + is addition and * is multiplication. *8*

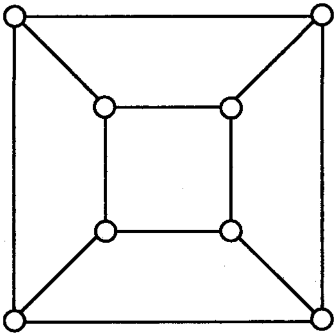
(b) Show that N, the set of natural numbers is a semigroup under the operation $x*y = \max(x, y)$. Is it a monoid ? *7*

2. (a) If $f : S \rightarrow T$ is a homomorphism from $< S, * >$ to $< T, + >$, and $g : T \rightarrow P$ is also a homomorphism from $< T, + >$ to $< P, \$ >$, then $gof : S \rightarrow P$ is a homomorphism from $< S, * >$ to $< P, \$ >$. *8*

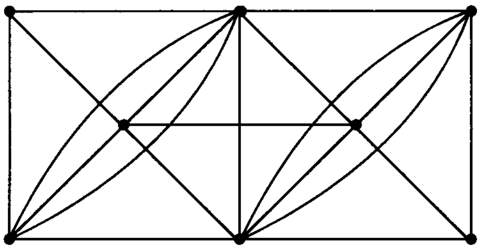
(b) Let $< A, * >$ be a group. Show that $(a*b)^{-1} = b^{-1}*a^{-1}$. *7*

SECTION II

3. (a) Define bipartite graph. Find whether the following graph is bipartite or not : *6*



(b) Find whether the following graph is planar or not. *8*



4. (a) Let a and b be numeric functions defined as follows :

$$a_r = \begin{cases} 1, & 0 \le r \le 2 \\ 0, & r \ge 3 \end{cases} \qquad b_r = \begin{cases} 0, & 0 \le r \le 1 \\ 1, & 2 \le r \le 5 \\ 2, & r \ge 6 \end{cases}$$

Find a+b, a*b, s^3a, $s^{-2}b$. *6*

(b) Write generalized pigeonhole principle. Use any form of pigeonhole principle to solve the given problem : *8*

   (i) Find minimum number of students in the class to be sure that three of them are born in the same month.

   (ii) Assume that there are 3 men and 5 women in a party. Show that if these people are lined up in a row, at least two women will be next to each other.

5. (a) Draw the Hasse diagram of the following sets under the partial ordering relation 'divides' and indicate, which are chains : *8*

   (i) {2, 4, 12, 24}

   (ii) {1, 3, 5, 15, 30}

(b) Let A = {6, 2, 1, 9}, B = {7, 3, 5}, C = {8, 4, 10}. Give the pictorial representation of the functions f and g. Obtain the composition of the following functions : $f : A \to B$, $g : B \to C$

where f = {(6, 7), (2, 3), (1, 5), (9, 3)}

g = {(7, 4), (3, 8), (5, 10)}. *6*

6. (a) Consider the following relation on {1, 2, 3, 4, 5, 6} : *3+3*

$$R = \{< i,\ j > \mid |i - j| = 2\}.$$

   (i) Write the set R.

   (ii) Is R

      (1) transitive ?

      (2) reflexive ?

      (3) symmetric ?

(b) Let A = {1, 2, 3} and B = {a, b, c, d}. In each case, state whether the given function (if defined) is injective, surjective, bijective. *2+2+2+2*

(i) $f = \{(1, a), (2, d), (3, b)\}$

(ii) $g = \{(1, a), (2, a), (3, d)\}$

(iii) $h = \{(1, a), (1, b), (2, d), (3, c)\}$

(iv) $j = \{ (1, a), (2, b)\}$

7. (a) Define partition of a set. Let A = {a, b, c, d}, P = { {a, b}, {c}, {d} } be the partition. Find the equivalence relation induced by P and construct its digraph. *6*

(b) Let A, B, C be arbitrary sets. Show that *4+4*

(i) $(A - B) - C = A - (B \cup C)$

(ii) $A \cup (B \cup C) = (A \cup B) \cup C$

8. (a) Prove by mathematical induction *8*

$$\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\ldots+\frac{1}{(3n-2)(3n+1)}=\frac{n}{3n+1}$$

(b) Define with examples : *3+3*

(i) Poset

(ii) Lattice