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MIA-010 (F2F)

00643

M.Sc. ACTUARIAL SCIENCE (MSCAS)

Term-End Examination

December, 2012

MIA-010 (F2F) : STATISTICAL METHOD

Time : 3 hours *Maximum Marks : 100*

Note : *In addition to this paper you should have available Actuarial table and your own electronic calculator.*

SECTION - A

There are **seven** questions each of 8 marks. Answer **any five.**

1. (a) Explain what is meant by a zero sum two person game. **2**

(b) X has a normal distribution with parameters. μ and σ^2. State an expression for $E(X^4)$. **4**

(c) The random variable X has the p.d.f. $f(x) = 2\lambda^2 (\lambda + x)^{-3}$, $x > 0$ where λ is a positive constant. Derive an expression for the probability that X exceeds $k\lambda$ $(k > 0)$. **2**

2. A generalised linear model has Binomial responses $Z_1.....Z_k$ with $E(Zi) = n\mu$, $var(Zi) = n\mu(\mu - 1)$ for $0<\mu<1$.

(a) Show that $Y_i = {}^{Z_i}\!/_{n}$ belongs to an exponential family. 4

(b) Identify the natural parameter and the canonical link function and drive the variance function. 4

3. The random variable X has the following discrete distribution. 8

$p(X = x) = p\ (1 - p)^{x-1}$ $x = 1, 2, ...$ where $0<p<1$. By deriving a formula for the moment generating function or otherwise. Show that the mean and variance of X are p^{-1} and $(1 - p)\ p^{-2}$ respectively.

4. (a) State the two conditions that must hold for a risk to be insurable. 3

(b) List five other criteria that would be considered desirable by a general insurer. 5

5. By first finding the values of ρ_1 and ρ_2 or otherwise find ϕ_2, the 2nd partial auto correlation, for the series. $X_n = \alpha X_{n-1} + \beta X_{n-2} + \epsilon_n$, expressing. Your answer in simplest form and also comment on your answer. 8

6. (a) Describe the difference between strictly stationary process and weakly stationary process. 3

(b) Show that the following bivariate time series process $(X_n\ Y_n)^t$, is weakly stationary : 5

$$X_n = 0.5\ X_{n-1} + 0.3\ Y_{n-1} + e_n^x$$

$$Y_n = 0.1\ X_{n-1} + 0.8\ Y_{n-1} + e_n^y$$

where e_n^x and e_n^y are two independent white noise processes.

7. The table below shows annual aggregate claim statistics for 3 risks over 4 years. Annual aggregate claims for risk i in year j are denoted by X_{ij}.

Risk	$\overline{X}_i = \frac{1}{4}\sum_{j=1}^{4} X_{ij}$	$S_i^2 = \frac{1}{3}\sum_{j=1}^{4}\left(X_{ij}-\overline{X}_i\right)^2$
1	2517	4121280
2	7814	7299175
3	2920	3814001

(a) Calculate the value of the credibility factor for Empirical Bayes Model 1. 4

(b) Using the numbers calculated in (a) to illustrate your answer describe the way in which the data affect the value of the credibility factor. 4

SECTION - B

There are *six* questions each of **15** marks. Answer *any four*.

8. (a) Under a special reinsurance arrangement a reinsurance agrees to pay an amount Y in respect of each claim X, arising from a certain risk where **10**

$$Y = \begin{cases} 0 & \text{if} \quad x \leq 1000 \\ x - 1000 & \text{if} \quad 1000 < x \leq 2000 \\ 1000 & \text{if} \quad x > 2000 \end{cases}$$

Given that X has a lognormal distribution with parameters $\mu = 5$, $\sigma^2 = 4$, find the reinsures's expected payment amount per original claim.

(b) Show that, given a random sample of size n, from a log $N(\mu, \sigma^2)$ distribution, if an uninformative prior is used for μ, then posterior distribution for **5**

$$\mu \text{ is } N\left(\frac{1}{n}\Sigma Wg\, x_i, \frac{\sigma^2}{n}\right).$$

9. The table below show the cumulative cost of incurred claims and the number of claims reported each year for a certain cohort of insurance policies. The claims are assumed to be fully run - off at the end of development year 2 cummulative cost of incurred claims.

A/Y	Development		
	0	1	2
0	288	634	893
1	465	980	
2	773		

The numbers of claim reported in each year

AY	Development		
	0	1	2
0	110	85	55
1	167	113	
2	285		

Given that the total amount paid in claims to date, relating to accident years 0, 1, 2 is Rs. 2750, calculate the outstanding claims reserve using the average cost per claim method. **15**

10. Claims occur as a poisson process with rate λ and individual claim size X follow an exp (β) distribution. The office premium includes a security loading θ_1. An individual *excess* of loss arrangement operates under which the reinsurer pays the excess of individual claims above an amount M in return for a premium equal to the reinsure's risk premium increased by a proportionate security loading θ_2. Derive and simplify as far as possible an equation satisfied by the adjustment co - efficient for the direct insurer. 15

Also use the approximation $e^x = 1 + x + \frac{x^2}{2}$ to find an approximate numerical value for the adjustment coefficient for the above example in the case where $\beta = 0.05$, $\theta_1 = 0.3$, $\theta_2 = 0.4$ and $M = 10$.

11. An insurer plans to issue 5000 one - year policies at the start of a year. For each policy the annual aggregate claims have a compound negative binomial distribution ; the negative binomial parameters are $k = 0.5$ and $p = 0.5$, and individual claim amounts, in Rupees, have a log normal distribution with parameters $\mu = 5.04$ and $\sigma = 1.15$. 15

The premium for each policy is 160 and is payable at the start of the year claims are assumed to be paid at the mid point of the year. Calculate the minimum annual rate of interest the insurer must earn throughout the year if the accumulation to the end of the year of premiums minus claims is to exceed 52,000 with probability 90%. You may assume that the distribution of total aggregate claims in the year may be approximated by a normal distribution.

12. (a) A stationary ARMA (1, 1) series is defined by $X_n = \alpha X_{n-1} + \epsilon_n + \beta\epsilon_{n-1}$, where $\epsilon t \sim N(0, \sigma^2)$ denotes the white noise. Show that P_k, the autocorrelation at log k. (k=1, 2, 3, ...) for this process is : **10**

$$P_k = \frac{(\alpha+\beta)\,(1+\alpha\beta)}{1+2\alpha\beta+\beta^2}\,\alpha^{k-1} \quad (k=1,\ 2,\ 3...)$$

Hence deduce formula for the auto-correlation function of an AR(1) and an MA(1) series.

(b) Define β and ∇. and show that : **5**

$$\nabla^d_{X_n} = \sum_{k=0}^{d} (-1)^k \binom{d}{k} X_{n-k}$$

$d = 0,\ 1,\ 2...$

13. (a) Explain, what is meant by I(0) and an I(1) process. 2

(b) Explain, what is meant by a "Prior distribution" a "posterior distribution" and a "loss function". 3

(c) What is meant by the Markov property for a Univariate random process $\{X_n\}_{n=1}^{\infty}$? 2

(d) The following pseudo - random numbers. Come from uniform distribution on the interval (0, 1). 8

0.667, 0.457, 0.671, 0.881, 0.911 use these to generate five pseudo - random number (rounded to nearest whole number) from the pareto distribution with p.d.f.

$$f(x) = \frac{44.194}{(50+x)^{3.5}} \quad (x > 0).$$
