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MIA-005 (F2F)

M.Sc. IN ACTUARIAL SCIENCE (MSCAS)

Term-End Examination

December, 2012

00213

MIA-005 (F2F) : STOCHASTIC MODELLING AND SURVIVAL MODELS

Time : 3 hours *Maximum Marks : 100*

Note : *In addition to this paper you should have available the ACTUARIAL table and your own electronic calculator.*

SECTION - A

(Answer *any five* questions) **8x5=40**

1. (a) List the benefits of modelling in actuarial work. 4

(b) Explain the main differences between a deterministic model and a stochastic model. 4

2. Let us consider a homogeneous Markov chain with state space $S = \{1, 2, 3\}$ and the corresponding transition matrix is given by :

$$P = \begin{pmatrix} 1/4 & 1/2 & 1/4 \\ 1/2 & 0 & 1/2 \\ 3/4 & 1/4 & 0 \end{pmatrix}$$

Using this information

(a) Calculate the three step transition matrix. 2

(b) Calculate for each of the following initial conditions, the probability that the chain will be in state 3 when it is observed at time $n=3$ given that : 3

(i) The chain is in state 1 at time zero and in state 2 at time 1.

(ii) The probabilities, of being in state 1, 2 and 3 at time zero, are given by $\frac{14}{31}$, $\frac{9}{31}$ and $\frac{8}{31}$ respectively.

(c) Comment on how the answer to (i) and (ii) in part (b) would change if the time of the observation was $n=300$ instead of $n=3$? 3

3. A credit-worthiness of debt issued by companies is assessed at the end of each year by a credit rating agency. The rating are A (the most credit-worthy), B and D (debt defaulted). Historic evidence supports the view that the credit rating of a debt can be modelled as a Markov chain with one-year transition matrix :

$$X = \begin{pmatrix} 0.92 & 0.05 & 0.03 \\ 0.05 & 0.85 & 0.1 \\ 0 & 0 & 1 \end{pmatrix}$$

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(a) Determine the probability that a company currently rated A will never be rated B in the future. 2

(b) (i) Calculate the second order transition probabilities of Markov chain. 2

(ii) Hence calculate the expected number of defaults within the next two years from a group of 100 companies, all initially rated A.

The manager of a portfolio investing in company debt follows a "downgrade trigger" strategy. Under this strategy, any debt in a company whose rating has fallen to B at the end of a year is sold and replaced with debt in an A-rated company.

(c) Calculate the expected number of defaults for this investment manager over the next two years, given that the portfolio initially consists of 100 A-rated bonds. 2

(d) Comment on the suggestion that the downgrade trigger strategy will improve the return on the portfolio. 2

4. (a) Define the following types of a stochastic process : 3

(i) a Poisson process

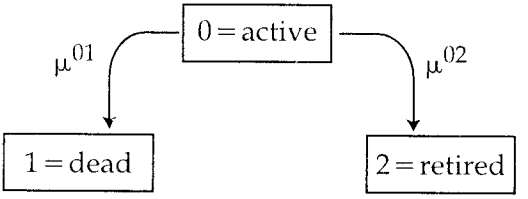
(ii) a compound Poisson process

(iii) a general random walk

(b) For each of the process in (a), state whether it operates in continuous or discrete time and whether it has a continuous or discrete state space. 2

(c) For each of the process in (a), describe one practical situation in which an actuary could use such a process to model a real world phenomenon. 3

5. Consider the two-decrement model, in which the transition intensities are constant. 8



Prove that

$$ {}_tP_x^{01} = \frac{\mu^{01}}{\mu^{01} + \mu^{02}} \left[1 - e^{-(\mu^{01} + \mu^{02})t}\right] $$

6. A mortality investigation covered the period 1 January 2007 to 1 January 2008. This is measured in years from 1 January 2007 and $P_x(t)$ denotes the number of lives at time t aged x last birthday. The following data were recorded for each x :

dx : number of deaths aged x next birthday.

$P_x(0)$ and $P_x(1)$.

(a) Obtain an expression for the initial exposed to risk in terms of the available census data that may be used to estimate the initial rate of mortality q_{x+f} stating your assumptions. 6

(b) Determine the value of f, stating any assumptions you make. 2

7. Discuss the advantages and disadvantages of using the graphical method of graduation, explaining under which circumstance it is appropriate to use it. 8

SECTION - B

(Answer *any four* questions) **15x4=60**

8. (a) (i) Prove that under Gompertz's Law, the probability of survival from age x to age $x+t$, ${}_tP_x$ is given by 3

$$ {}_tP_x = \left[\exp\left(\frac{-B}{\ln c}\right)\right]^{C^x(C^t-1)} $$

For a certain population, estimates of survival probabilities are available as follows :

${}_1P_{50} = 0.995$

${}_2P_{50} = 0.989$

(ii) Calculate values of B and C consistent with these observation. 3

(iii) Comment on the calculation in (ii) compared with the usual process for estimating the parameters from a set of crude mortality rates. 3

(b) (i) Write down a formula for ${}_tq_x$ $(0 \le t \le 1)$ under each of the following assumptions : 2

(A) uniform distribution of deaths.

(B) constant force of mortality.

(C) the Balducci assumption.

(ii) Calculate ${}_{0.5}P_{60}$ to six decimal places under each assumption given $q_{60} = 0.05$. 2

(iii) Comment on the relative magnitude of your answer in part (b). 2

9. An investigation took place into the mortality of pensioners. The investigation began on 1 January 2003 and ended on 1 January 2004. The table below gives the data collected in this investigation for 8 lives.

Date of birth	Date of entry into observation	Date of exit from the observation	Whether or not exit was due to death (1) or other reason (0)
1 April 1932	1 January 2003	1 January 2004	0
1 October 1932	1 January 2003	1 January 2004	0
1 November 1932	1 March 2003	1 September 2003	1
1 January 1933	1 March 2003	1 June 2003	1
1 January 1933	1 June 2003	1 September 2004	0
1 June 1933	1 January 2003	1 January 2004	0
1 March 1933	1 September 2003	1 January 2004	0
1 October 1933	1 June 2003	1 January 2004	0

The force of mortality, μ_{70}, between exact ages 70 and 71 is assumed to be constant.

(a) (i) Estimate the constant force of mortality μ_{70}, using a two-state model and the data for the 8 lives in the table. 7

(ii) Hence or otherwise estimate q_{70}.

(b) Show that the maximum likelihood estimate of the constant force, μ_{70}, using a Poisson model of mortality is the same as the estimate using the two-state model. 5

(c) Outline the difference between the two-state model and the poisson model when used to estimate transition rates. 3

10. (a) Compare the advantages and disadvantages of fully parametric models and the Cox-regression model for assessing the impact of covariates on survival. 3

You have been asked to investigate the impact of a set of covariates, including age, sex, smoking, region of residence, educational attainment and amount of exercise undertaken, on the risk of heart attack. Data are available from a prospective study which followed a set of several thousand persons from an initial interview until their first heart attack, or until their death from a cause other than a heart attack, or until 10 years had elapsed since the initial interview (which ever of these occurred first).

(b) State the types of censoring present in this study and explain how each arises. 2

(c) Describe a criterion which would allow you to select those covariates which have a statistically significant effect on the risk of heart attack, when controlling the other covariates of the model. 4

Suppose your final model is a Cox model which has three covariates : age (measured in age last birthday minus 50 at the initial interview), sex (male = 0, female = 1) and smoking (non-smoker = 0, smoker = 1), and that the estimated parameters are :

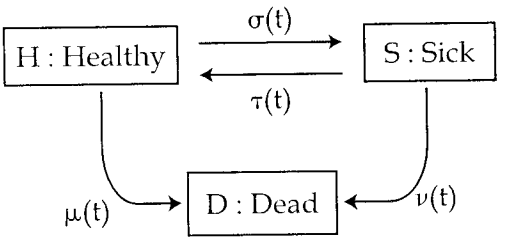
Age	0.01
Sex	− 0.4
Smoking	0.5
Sex X smoking	− 0.25

Where "sex X smoking" is an additional covariate formed by multiplying the two covariates "sex" and "smoking".

(d) Describe the final model's estimate of the effect of sex and of smoking behaviour on the risk of heart attack. 3

(e) Use the results of the model to determine how old a female smoker must be at the initial interview to have the same risk of heart attack as a male non-smoker aged 50 years at the initial interview. 3

11. In a Markov jump process model of sickness and death there are three states : healthy, sick and dead, {H, S, D}. The transition graph is shown below. Denote the state of the process at time t by X_t.



(a) Give the transition rates in the form of a matrix. 2

(b) Define the residual holding time Rs. 1

(c) If someone is sick at time S, give an integral expression for the probability they remain sick for a further period of atleast ω. 2

(d) State the probability density function of Rs (given $X_S = S$) 2

(e) If we are told that a transition from H has just taken place, give an expression for the probability that it was to S. 2

(f) Give the backward Kolmogorov equation for $P_{SD}(s, t)$, the probability that an individual who is sick at time S is dead at time t. 6

12. (a) State three different methods of graduating raw mortality data and for each method give an example of a situation when the method would be appropriate. 3

A life insurance company last priced its whole of life contract 30 years ago using a standard mortality table. The company wishes to establish whether recent mortality experience in the port folio of business is in line with the pricing basis. These are the data :

Recent Experiences

Age last birthday	Exposed to Risk during 2009	Deaths during 2009
50	2381	16
51	3177	21
52	3460	22
53	1955	15
54	3122	24
55	3485	29
56	2781	26
57	3150	31
58	3651	39
59	3991	48

Extract from the standard table used for pricing the product

x	Number of survivors to age x
50	32669
51	32513
52	32338
53	32143
54	31926
55	31685
56	31417
57	31121
58	30795
59	30435
60	30039

(b) Test the goodness of fit of these data with the pricing basis and comment on your results. **8**

(c) (i) State with reasons, one further test which you would deem appropriate to perform on these data. **4**

(ii) Carry out that test.

13. You have been asked to advise a sports magazine, as a consultant statistician. You have been asked to investigate the hypothesis that football managers in the country X are dismissed more quickly than those in country Y premier league. Each league has twenty teams, each with one manager. During the season the following events happened (at the end of the months indicated) to the twenty managers who started :

Month	X	Y
1	One dismissed	One died
3	-	One left of his own accord
5	One left of his own accord	One dismissed
6	Two dismissed	One left of her own accord
8	One died	Two dismissed
11	Two dismissed	One dismissed

Hence these were thirteen of the original twenty managers still employed by the same club at the end of the season, for each of the two leagues.

(a) Calculate the Kaplan-Meier estimate of the distribution function and its approximate variance for each league separately. **12**

(b) Comment on the hypothesis that country X managers dismissed more quickly than those for country Y. **3**