No. of Printed Pages : 3 **MMTE-001**

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)
Term-End Examination

00260

December, 2012
MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*
Weightage : 50%

Note : *Question no. 1 is **compulsory**. Answer **any four** of the remaining six (2 to 7) Calculating devices are **not** allowed.*

1. State whether each of the following statements is ***true*** or ***false***. Justify your answer with appropriate arguments or illustrations. **2x5=10**

(a) C_5 is a self - complementary graph.

(b) In any graph, number of vertices odd degree is even.

(c) Number of perfect matchings in the complete graph k_{2n} is (2n)!

(d) For any graph G, $\chi(G) \geq \omega(G)$

(e) There exist graphs isomorphic to their own duals.

2. (a) Prove that a graph in which each vertex has degree at least two contains a cycle. **3**

(b) Write the chraracterization of the center of a tree. **3**

(c) Define interval graphs and prove that $\chi(G) = \omega(G)$ for an interval graph G. **4**

3. (a) Draw a plane embedding of a maximal planar graph with five vertices. Draw its dual also. 3+2

For a simple graph G.

(b) Prove that $K(G) \leq K'(G) \leq \delta(G)$. 5

4. (a) State Hall's theorem, three girls know 4 boys, as given in table. 6

Girls	Boys known
(a)	W, X, Y, Z
(b)	Y, Z
(c)	X, Z

draw the bipartite graph and check Hall's condition.

(b) Define connected graph. If u and v are the only odd vertices in a graph G, prove that G contains a u-v path. 4

5. (a) Compute the radius and diameter of the complete graph k_n and the complete bipartite graph $k_{n,n}$ 4

(b) There are five cities in a network. The cost of constructing a road directly between i^{th} and j^{th} city is the $(i, j)^{th}$ entry in the matrix. 6

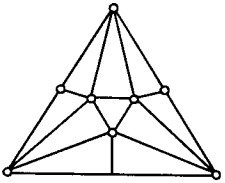
$$\begin{pmatrix} 0 & 3 & 5 & 11 & 9 \\ 3 & 0 & 3 & 9 & 8 \\ 5 & 3 & 0 & \infty & 10 \\ 11 & 9 & \infty & 0 & 7 \\ 9 & 8 & 10 & 7 & 0 \end{pmatrix}$$

An infinite entry indicates the impossibility of constructing a road due to geographical reasons. Draw a graph model of the system and use Kruskal's algorithm to determine the least cost of making all the cities reachable from each other.

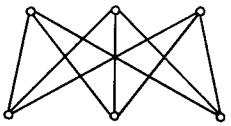
6. (a) If $\tau(G)$ denotes the number of spanning trees in graph G and if e is a non - loop edge in G, then prove that 4

$\tau(G) = \tau(G - e) + \tau(G. e)$.

(b) Decide which of the following graphs are Eulerian or Hamiltonian, or both ; (Give reasons) 6



(a)



(b)

7. (a) Prove that every component of the symmetric difference of two matchings is a path or an even cycle. 3

(b) Show that the minimum degree in a k-critical graph is at least k-1. 3

(c) Obtain an expression for the chromatic number of the Cartesian product of two given graphs in terms of the chromatic numbers of the given graphs. 4