No. of Printed Pages : **6**

MMT-009

M.Sc. MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE (MACS)

Term-End Examination 00611

December, 2012

MMT-009 : MATHEMATICAL MODELLING

Time : 1½ hours

Maximum Marks : 25

(Weightage : 70%)

Note : *Do **any five** questions. Use of calculator is **not** allowed.*

1. (a) Find a linear demand equation that best fits the following data, and use it to predict annual sales of homes priced at Rs. 14,00,000. **3**

x =price (lakhs of Rs.)	16	18	20	22	24	26	28
y =sales of new homes this year	126	103	82	75	82	40	20

(b) Give one example each from the real world for the following along with justification for your example. **2**

(i) A non-linear model

(ii) A stochastic model

2. (a) The deviation g(t) of a patients blood glucose concentration from its optimal concentration satisfies the differential equation. $4\frac{d^2g}{dt^2} + 8\alpha\frac{dg}{dt} + (2\alpha)^2 g = 0$ for α, a positive constant, immediately after the patient fully absorbs a large amount of glucose. The time t is measured in minutes. Identify the type (overdamped, underdamped or critically damped) of this differential equation. Find the condition on α for which the patient is normal. 2½

(b) The growth of a population is proportional to the population and is restricted by the lack of availability of resources like food, space, which can be modelled as proportional to the square of the population itself. 2½

(i) Model this process

(ii) Solve the resulting equation

(iii) Find the long term behaviour of the population.

3. Consider the discrete time population model given by $N_{t+1} = \dfrac{r N_t}{a + \left(\dfrac{N_t}{K}\right)^b}$, for a population N_t, where K is the carrying capacity of the population, r is the intrinsic growth rate, a and b are positive parameters where $a > r$. Determine the non-negative steady state and discuss the linear stability of the model. Also find the first bifurcation value of the parameter r. 5

4. Do the stability analysis of the following interacting system of species under the effect of toxicant, when the concentration of the toxicant in the environment is assumed to be constant. 5

$$\frac{dN_1}{dt} = r_1 N_1 - \alpha_1 N_1 N_2 - d_1 C_0 N_1$$

$$\frac{dN_2}{dt} = r_2 N_2 - \alpha_2 N_1 N_2 - d_2 V_0 N_2$$

$$\frac{dC_0}{dt} = k_1 P - g_1 C_0 - M_1 C_0$$

$$\frac{dV_0}{dt} = k_2 P - g_2 V_0 - M_2 V_0$$

under the initial conditions $N_1(0) = N_{10}$, $N_2(0) = N_{20}$, $C_0(0) = 0$, $V_0\ (0) = 0$. The variables and parameter notation in the above system of equation are $N_1(t)$, $N_2(t)$ = Densities of two different populations.

$C_0(t)$ = Concentration of the toxicant in the individuals of the population $N_1(t)$.

$V_0(t)$ = Concentration of the toxicant in the individuals of the population $N_2(t)$

P = Concentration of toxicant in the environment and is constant, r_1 and r_2 are the birth rates, α_1, α_2 are the predation rates, d_1 is the death rate due to C_0, d_2 is the death rate due to V_0, k_1, k_2 are uptake rates, g_1, g_2 are loss rates ; m_1, m_2 are depuration rates. Here r_1, r_2, α_1, α_2, d_1, d_2, k_1, k_2, g_1, g_2, m_1, m_2, and P are all positive constants.

5. (a) In a tumour region, the control parameters of growth and decay are in the ratio 2 : 1. Emigration occurs at a constant rate of 3×10^3 cells per month. There is decay of 35 cells every month. Use these assumptions to formulate the logistic model of tumour size. Solve the formulated equation if the initial size of the tumour is 5×10^6 cells. When does the proliferation of tumour cells stabilize for this model ? 2½

(b) If the standard deviation σ_p of the portfolio $P = (w_1, w_2)$ is minimum then for $\sigma_1 = \sigma_2$ show that $w_1 = w_2 = 0.5$. w_1 and w_2 are the portfolio weights and σ_1 and σ_2 be the standard deviations of the two securities 1 and 2 respectively. 2½

6. (a) How are the returns on the two securities A and B related ? When. 2

(i) The covariance between A and B is positive

(ii) There is zero correlation between A and B.

(b) A company has three plants A, B and C and four warehouses P, Q, R, S. Goods have to be transported from A, B and C to P, Q, R and S. The transportation cost per unit from. 3

A to P, Q, R, S are 21, 16, 25, 13

B to P, Q, R, S are 17, 18, 14, 23

C to P, Q, R, S are 32, 17, 18, 41 respectively

Capacities of the plants are 11, 13, 19 respectively and the requirements of the warehouses are 6, 10, 12, 15 respectively. The initial basic feasible solution of the transportation problem are given as follows :

11 for the cell (A, S) ;
6 for the cell (B, P) ;
3 for the cell (B, R) ;
4 for the cell (B, S) ;
10 for the cell (C, Q) ;
9 for the cell (C, R).

Determine how the units can be transported to warehouses so that the transportation cost is minimum.