# M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

## Term-End Examination

## December, 2012 00351

## MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

*Time : 2 hours* *Maximum Marks : 50*

*(Weightage : 50%)*

**Note :** *Question No. 1 is **compulsory**. Do **any four** questions out of question nos. 2 to 7. Use of calculator is **not** allowed.*

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**1.** State whether the following statements are ***true*** or ***false***. Justify your answer with the help of a short proof or a counter example. **2x5=10**

(a) The series $\sum_{n=0}^{\infty} n^n x^n$ converges for all non-zero $x$.

(b) The fourth order Runge-Kutta method is used to solve initial-value-problems

$$y' = -200y, \ y(0) = 1.$$

The value of h, so that the method produces stable results, is h < 0.015.

(c) the five-point finite difference formula for Poisson Equation $\nabla_u^2 = G(x, y)$ is of order $O(h^2)$.

(d) $L\left[\int_0^t e^{-3z} \cos(1-z)\, dz\right] = \frac{1}{s+3} \cdot \frac{s}{s^2+1}$.

(e) The Green's function does not exist for the boundary value problem

$y''(x) = 0$, $y(0) = y(1)$ and $y'(0) = y'(1)$.

2. (a) Find $L(\sin \sqrt{t})$, where $L$ denote Laplace transform. 5

Deduce the value of $L\left[\frac{\cos \sqrt{t}}{\sqrt{t}}\right]$.

(b) The heat conduction equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ 5

is approximated using two level explicit method. Investigate the stability of the method using the Von Neumann stability criteria. Also determine the truncation error of the method.

3. (a) Find a series solution about $x=0$ of the differential equation 6

$$4\,(x^4-x^2)\,y''+8\,x^3\,y'-y=0.$$

(b) Apply fourth order Runge-Kutta method to solve the initial-value problem. 4

$\frac{dy}{dx}=x-2y,\ y(0)=1$ to obtain $y$ for $x=0.1$ and $x=0.2$.

4. (a) Construct Green's function for the differential equation. 5

$$xy''+y'=0,\ 0<x<l$$

under the condition that $y\,(0)$ is bounded and $y\,(l)=0$.

(b) Find any one element in the solution of boundary value problem 5

$$\nabla^2 u = x+y,\ 0\le x\le 1,\ 0\le y\le 1$$

$u=\frac{1}{6}\,(x^3+y^3)$, on the boundary, using the Galerkin's method with rectangular elements and one internal node $(h=\frac{1}{2})$.

5. (a) Consider two point charges of equal magnitude q, but of opposite sign. These charges are placed in a polar coordinate system. Show that the potential U at any point P becomes. 5

$$U = \frac{2q}{r}\left[ P_1(\cos\theta)\frac{a}{r} + P_3(\cos\theta)\left(\frac{a}{r}\right)^3 + \ldots\right],$$

where $P_n(x)$ is Legender's polynomial of order n. 2a is the distance between two point charges and (r, θ) are the coordinates of the point P, where r > a.

(b) Find the Fourier sine transform of the function. 3

$$f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$$

(c) What is the difference between explicit single-step method and implicit single step method for solving IVP 2

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0.$$

6. (a) Solve boundary value problem 5

$$y'' = 6y^2, \; y(0) = 4, \; y\left(\frac{1}{2}\right) = 1.$$

using second order finite difference method

with $h = \frac{1}{4}$.

(b) Using Fourier transforms, determine the solution of the equation 5

$$\frac{\partial^4 z}{\partial x^4} + \frac{\partial^1 z}{\partial x^1} = 0, \left(-\infty < x < \infty, \ y \geqslant 0\right),$$

satisfying the conditions (i) z and its derivatives tend to zero as $x \rightarrow \pm \infty$

(ii) $z = f(x), \ \frac{\partial z}{\partial y} = \partial$ on $y = 0$.

7. (a) Solve the initial value problem $y^1 = -2xy^2$, $y(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$ using the predictor - corrector method 4

$$P : y_{k+1} = y_k + \frac{h}{2}\left(3\, y_k^1 - y_{k-1}^1\right)$$

$$C : y_{k+1} = y_k + \frac{h}{2}\left(y_{k+1}^1 + y_k^1\right).$$

Perform two corrector iterations per step.

Use the exact solution $y(x) = \frac{1}{1+x^2}$ to obtain the starting value.

(b) When does a finite difference method said to be consistent ? Discuss the consistency of the Laasonen method. 3

(c) Using Laplace transform, solve 3

$$y'' + y = 0,\ y\ (0) = 1,\ y'(0) = -2$$

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