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MMT-006

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

December, 2012

MMT-006 : FUNCTIONAL ANALYSIS

Time : 2 hours | *Maximum Marks : 50*
Weightage 70%

Note : *Question number **1** is compulsory. Attempt **any four** from the remaining questions.*

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1. State whether the following statements are True or False. Justify with a short proof or a counter example. **5x2=10**

(a) If a linear subspace Y of a normed linear space X has non-empty interior, then $Y = X$.

(b) Every linear map, between normed linear spaces, with a closed graph is continuous.

(c) A Hilbert space is necessarily strictly convex.

(d) The space l^1 is reflexive.

(e) Every bounded linear functional on a normed linear space is compact.

2. (a) Let X be a normed space and Y be a closed subspace of X. Define the quotient norm on the space X/Y. Verify that X/Y is a normed space under this norm. **4**

(b) Prove that the dual space of l^{∞} is not isometric to l^1 but contains a subspace isometric to l^1. **6**

3. (a) State the projection theorem for Hilbert spaces. Use it to prove the following - if M is a closed subspace of a Hilbert space H and $x_0 \in H$, $x_0 \notin M$, then there is a linear functional $f \in H^1$ such that $\| f \| = 1$, $f(M) = 0$, $f(x_0) = d(x_0, M)$. **6**

(b) prove that a normed linear space X is complete if every absolutely convergent series in X converges in X. **4**

4. (a) If H is a Hilbert space and $A \in BL(H)$ is positive, prove that $A + 2I$ is invertible. **3**

(b) Prove that any linear map between finite dimensional normed linear spaces is continuous. **3**

(c) Let T be the linear functional defined on C [a, b] (with sup norm) by **4**

$$T(x) = \int_a^b x(t)\,dt \quad \text{for } x \in C[a, b]$$

Find $\| T \|$

5. (a) State the Hahn - Banach Extension theorem. Illustrate with an example. 4

(b) Let X, Y be normed linear spaces. On X x Y define $\| (x,y) \|_1 = \| x \|_x + \| y \|_y$ and $\| (x,y) \|_\infty = \max (\|x\|_x, \|y\|_y)$. Prove that each defines a norm on X x Y and that they are equivalent. 4

(c) Determine $\{e_1, e_2, e_3, e_4\}^\perp$ in l^2. 2

6. (a) Give an example of an operator A on $(C[0,1], \| \|_\infty)$ with $\sigma(A) = [0,1]$. 4

(b) If X_0 is a dense subspace of a normed linear space X show that $X_0' \simeq X'$. 3

(c) Prove that the adjoint of a compact operator is compact. 3

7. (a) Show how to obtain the open mapping theorem from the closed graph theorem. 7

(b) Ortho normalise the sequence (1,0,0...), (1,1,0,0, ...), (1,1,1,0,0, ...), ... in l^2. 3
