MCA(Revised)

## Term-End Examination

December, 2012

## MCS-013 : DISCRETE MATHEMATICS

Time: 2 hours Maximum Marks : 50
Note: Q. No. 1 is compulsory.
Attempt any three from the rest.

1. (a) Prove the following equivalence

$$
\sim\left(\exists_{x} \sim \mathrm{P}(x)\right) \equiv \forall_{x} \mathrm{P}(x)
$$

(b) Use proof by contradiction to prove that $\sqrt{2}$ is irrational.
(c) Simplify the following boolean expression 3 $\left(a^{\prime} \wedge b^{\prime} \wedge c^{\prime}\right) \vee\left(a^{\prime} \wedge b^{\prime} \wedge c\right) \vee\left(a \wedge b^{\prime} \wedge c^{\prime}\right) \vee\left(a^{\prime} \wedge b \wedge c^{\prime}\right)$
(d) Use Venn diagram to show the following 3 set operation.
(i) $\overline{\mathrm{A}}$
(ii) $A \cup(B \cap C)$
(iii) $A \cap(B \cup C)$
(e) Why is $y^{2}=x$ not a function?
(f) An urn contains 15 balls, 8 of which are red 2 and 7 are black. In how many ways 5 balls can be drawn such that
(i) all 5 are red.
(ii) 3 are red and 2 are black.
(g) In a survey of 260 college students following data was obtained.
64 had taken mathematics cource
94 had taken computer science.
58 had taken business studies.
28 had taken both mathematics and business studies
26 had taken both mathematics and computer science
22 had taken both computer science and business studies
14 had taken all three types of courses . What is the probability that a student chosen at random had not taken any course ?
2. (a) Construct truth table to check whether the following is a tautology, contigency or absurdity.
(i) $\mathrm{p} \wedge \sim \mathrm{p}$
(ii) $\mathrm{q} \rightarrow(\mathrm{q} \rightarrow \mathrm{p})$
(b) If $\mathrm{p} \rightarrow \mathrm{q}$ is false, what is the truth value of $(\sim(\mathrm{p} \wedge \mathrm{q})) \rightarrow \mathrm{q}$ ? Explain.
(c) Write the contrapositive \& converse of the statement :
If it rains then I will get wet.
(d) Prove by mathematical induction

$$
1^{3}+2^{3}+3^{3}+--+n^{3}=\frac{n^{2}(n+1)^{2}}{4}
$$

3. (a) For the following circuit write the boolean expression

(b) Make the circuit for the following boolean expression using logic gates
$\left(\left(x_{1} \wedge x_{2}\right)^{\prime} \vee\left(x_{3} \vee x_{4}\right)\right) \wedge\left(x_{1} \wedge x_{3}\right)^{\prime} \wedge\left(x_{2} \wedge x_{4}^{\prime}\right)$
(c) For the following truth table write DNF and CNF.

| $x_{1}$ | $x_{2}$ | $x_{3}$ | $\mathrm{f}\left(x_{1}, x_{2}, x_{3}\right)$ |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

4. (a) Explain the following types of relations with 4 the help of suitable examples.
(i) Reflexive
(ii) Antisymmetric
(iii) Transitive
(iv) Equivalence
(b) Let:

$$
f=\left(\begin{array}{llll}
1 & 2 & 3 & 4 \\
2 & 4 & 1 & 3
\end{array}\right) \text { and } g=\left(\begin{array}{llll}
1 & 2 & 3 & 4 \\
3 & 2 & 1 & 4
\end{array}\right)
$$

Find fog and gof.
(c) In how many ways 6 men and 6 women 2 can sit alternately in a row.
(d) "If a function is not one to one then it is not invertible." Explain.
5. (a) Prove that ${ }^{n+1} C_{r}={ }^{n} C_{r}+{ }^{n} C_{r-1}$. 3
(b) A and B are two mutually exclusive events 2 such that $P(A)=0.3$ and $P(B)=0.6$. What is the probability that
(i) B does not occur?
(ii) A or B occurs.
(c) If there be a set $A$ partitioned into $n$ number 3 of subsets. Show that the largest subset contains at least $\frac{|A|}{n}$ number of elements.
(d) How many 7 digits numbers are composed 2 of only odd digits ?

