No. of Printed Pages : **3** **MMTE-005**

M.Sc. MASTER IN MATHEMATICS WITH APPLICATIONS TO COMPUTER SCIENCE (MACS)

Term-End Examination

December, 2013

MMTE-005 : CODING THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage 50%)

Note : (i) *Answer **any five** questions from questions **1** to **6**.*

(ii) *Calculators are **not** allowed.*

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1. (a) Let e be the binary code given by the generator matrix **4**

$$G=\begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Find the dual of e and its minimum distance.

(b) Define a perfect code. Verify whether the binary repetition code **3**

$C=\begin{cases} 0 & 0 & . & . & 0 \\ 1 & 1 & . & . & 1 \end{cases}$ of block length n where

n is odd, is a perfect code.

(c) Check whether the polynomial **3**

$x^7 + x + 1 \in \mathbf{F}_2[x]$ is primitive.

2. (a) Let $f(x) = x^3 + x^2 + x + 1$ and $g(x) = x^4 + x^3 + x^2 + 1$. 5

Let $h(x) = \gcd(f(x), g(x))$ Find $a(x), b(x) \in \mathbf{F}_2[x]$ such that $a(x)f(x) + b(x)g(x) = h(x)$.

(b) Construct the generating idempotents of the duadic codes of length 11 over $\mathbf{F}_3$. 5

3. (a) (i) Define cyclic code and give an example. 2

(ii) Find the 2- cyclotomic cosets modulo 9 and factors of $x^9 - 1$ 3

(b) Consider the binary BCH[15,5] error correcting code with the generator polynomial. 5

$g(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$.

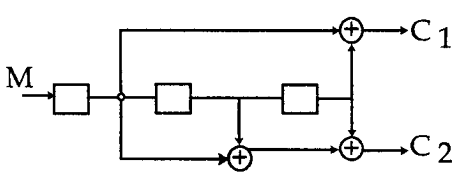
The code word received is $x^3 + x^5$ Decode the code word using Petersen-Gorenstein-Zierler algorithm.

4. (a) Find the weight enumerator of a self dual [12,6,6] ternary code. 5

(b) Find the code generated by the matrix 5

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 2 \end{bmatrix} \text{ over } \mathbf{L}_4.$$

5. (a) Find the convolutional code for the message 1011001, the convolutional encoder is given below 5



(b) Explain the two way APP decoding algorithm of turbo codes. 5

6. (a) Write down the generator matrix of a[15, 11] Hamming code. How many errors can it detect and how many errors can it correct. 5

(b) Let $x, y \in \mathbf{F}_2^n$. Show that 2

$\omega t(x+y) = \omega t(x) + \omega t(y) - 2\omega t(x \cap y)$

(c) Let e be the [6,3] binary code with the generator matrix 3

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(i) Prove that e is not self orthogonal.

(ii) Show that the code words whose weights are divisible 4 do not form a sub code of e.
