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MMT-007

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

December, 2013

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 hours *Maximum Marks : 50*
(Weightage : 50%)

Note : *Question No.1 is* ***compulsory****. Do* ***any four*** *questions out of question nos.2 to 7.* ***All*** *computations may be kept to 3 decimal places. Use of calculators is not* ***allowed****.*

1. State whether the following statements are **true** or **false**. Justify your answer with the help of a short proof or a counter example : **2x5=10**

(a) For the equation $x^2y'' + y' + (x^2 - P^2)y = 0$, $x = 0$ is a regular singular point.

(b) If α, β are two roots of $J_o(x) = 0$ and $\alpha^2 \neq \beta^2$, then $\int_0^1 xJ_o(\alpha x)J_o(\beta x)dx = 1$.

(c) The inverse Fourier transform $F^{-1}\left(\frac{1}{\alpha^2 + 4\alpha + 13}\right) = \frac{1}{6}e^{-3(|x| + ix)}$.

(d) The interval of absolute stability of the Runge-Kutta method

$$y_{i+1} = y_i + \frac{1}{4}(k_1 + 3k_2) \text{ where}$$

$$k_1 = hf(x_i, y_i)$$

$$k_2 = hf\left(x_i + \frac{2h}{3}, y_i + \frac{2k_1}{3}\right)$$

is $]-2, 0[$.

(e) The method

$\frac{\partial u}{\partial x} = \frac{1}{h}\left[u(x+h), y(t) - u(x, y(t))\right]$ is of order 2.

2. (a) Using Laplace transform, solve the following initial value problem : 4

$y'' + 25y = 10\cos 5x,\ y(0) = 2,\ y'(0) = 0.$

(b) Show that 3

$1 = 2\sum_{\alpha} \frac{J_o(\alpha x)}{\alpha J_1(\alpha)}$, $0 < x < 1$, where the summation is taken over all positive zeros of $J_o(x)$.

(c) Find the Fourier transform of e^{-4x^2}. 3

3. (a) Find the power series solution near $x = -1$ of the differential equation $(x^2 - 1)y'' + (5x + 4)y' + 4y = 0$. 6

(b) Derive the constants in the method $y_{i+1} = ay_i + h[b_o y'_{i+1} + b_1 y'_i + b_2 y'_{i-1}]$ for solving the initial value problem $y' = f(x, y)$, $y(x_o) = y_o$. Find the truncation error and the order of the method. 4

4. (a) Solve the initial value problem $y' = -2xy^2$, $y(0)=1$ in the interval [0, 0.4] using the predictor-corrector method: 5

$$P : y_{i+1} = y_i + \frac{h}{2}(3f_i - f_{i-1})$$

$$C : y_{i+1} = y_i + \frac{h}{2}(f_{i+1} - f_i)$$

with $h = 0.2$. Calculate the starting value starting value using second order Taylor series method with the same stepLength. Perform two corrector iterations per step.

(b) Find the Laplace transform of $x^{3/2}$. 2

(c) Find Laplace inverse of $\cot^{-1} s$. 3

5. (a) Find the solution of the initial-boundary value problem 5

$u_t = u_{xx}$, $0 \le x \le 1$, $t > 0$

$$u(x,0) = \begin{cases} 2x, & 0 \le x \le \frac{1}{2} \\ 2(1-x), & \frac{1}{2} \le x \le 1 \end{cases}$$ and

$u(0, t) = o = u(1, t)$, $t > 0$ Using Crank-Nicholson method with $\lambda = \frac{1}{2}$. Assume $h = \frac{1}{4}$ and integrate for one time level.

(b) Find the general solution of $y'' + (k/x^2)y = 0$, $x > 0$ for all possible values of the constant k. 5

6. (a) Expand $f(x) = x^3 - 3x^2 + 2x$ in a series of the form $\sum_{n=0}^{\infty} a_n H_n(x)$, where H_n are Hermite polynomials. 3

(b) Find the solution of the initial boundary value problem $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 \leq x \leq 1$ 4

$u(x,0) = \sin \pi x, 0 \leq x \leq 1,$

$\frac{\partial u}{\partial t}(x,0) = 0, 0 \leq x \leq 1$

$u(0, t) = u(1, t) = 0$, $t > 0$ by using second order explicit method with $h = \frac{1}{4}$, $r = \frac{1}{3}$

Integrate for one time step.

(c) Show that $\int_{-1}^{1} x P_n(x) P_{n-1}(x) dx = \frac{2n}{4n^2 - 1}$ 3

where $P_n(x)$ is n^{th} degree Legendre Polynomial.

7. (a) Solve the boundary value problem 7

$y'' - 3y' + 2y = 2$

$y(0) - y'(0) = -1$

$y(1) + y'(1) = 1$

Using the second order finite difference method with $h = \frac{1}{2}$.

(b) Derive the standard five point formula for one-dimensional Poisson equation with spacing h and k in x and y directions respectively. 3