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MMT-005

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M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

Term-End Examination

December, 2013

MMT-005 : COMPLEX ANALYSIS

Time : 1½ hours *Maximum Marks : 25*

Note : *Question No. **1** is **compulsory**. Attempt **any three other** questions. Use of calculator is **not** allowed.*

1. State giving reasons whether the following statements are **true** or **false** : **5x2=10**

(a) If T be a linear fractional transformation such that $T(0)=0$ and $T(\infty)=\infty$, then $T(z)=\alpha.z$ for same non-zero complex number α.

(b) $\lim_{z\to 0}\left(\frac{\bar{z}}{z}\right)=1$.

(c) If $f(z)=\log z$ then z=0 is an isolated singular point of $f(z)$.

(d) $\oint_c \frac{4z}{4z^2-4z+1}\,dz=2\pi i$, where c is the circle $|z|=1$.

(e) $f(z)=\frac{2z-1}{2-z}$ has a unique point of maximum modulus in $D=\{z:|z|\le 1\}$.

2. (a) Let $f(z)=z.Re(z)$. Determine where $f'(z)$ exists and find its value. 3

(b) Let $f(z)$ be analytic in a domain D. Prove that $f(z)$ is constant if $|f(z)|$ is constant. 2

3. (a) Find all the solutions of the equation $e^z=1$. 2

(b) Let C be the circle $|z|=3$ described in the positive sense. If $g(w)=\int_c \frac{e^z+z}{(z-w)^2}dz$, $z\in c$, then find g(2). What is the value of g(4) ? 3

4. (a) Let $f(z)$ be an entire function such that $f(0)=0$ and $|f'(z)|\leq|z|$ for all z. Prove that $f(z)=\alpha.z^2$, where α is same fixed complex number. 3

(b) Find the residue of $f(z)=\frac{1}{4z-z^2}$ at $z=0$ by expanding $f(z)$ in the Laurent Series in the region $0<|z|<4$. Hence show that $\int_c f(z)dz=\frac{\pi i}{2}$ for any positively oriented Circle $c=|z|=R<4$. 2

5. Evaluate $\int_0^{\pi} \frac{d\theta}{2+\cos\theta}$. 5