No. of Printed Pages : 4 **MMT-004**

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

December, 2013

MMT-004 : REAL ANALYSIS

Time : 2 hours *Maximum Marks : 50*

Weightage : 70%

Note : Question No. **1** *is* ***compulsory****. Do* ***any four*** *questions out of questions no.* 2 *to* 7.

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1. State, whether the following statements are **True or False**. Give reasons for your answers. **5x2=10**

   (a) The function $d : R \times R \to R$, defined by $d(x, y) = 8(x - y)$ is a metric on **R**.

   (b) The sequence $\{f_n\}$ in C [0, 1], given by $f_n(x) = x^n$, $x \in [0, 1]$ is convergent in C [0, 1] with the metric defined by

   $$d(f, g) = \int_0^1 |f(t) - g(t)|\, dt,$$ where the integral is Riemann integral.

   (c) The outer measure of the set of natural numbers is zero.

   (d) The function $f : R \to R^2$, defined by $f(x) = \left(x, x \sin\frac{1}{x}\right)$ is not differentiable at $x = 0$.

   (e) The set $\{n \in Z : -10 \le n \le 10\}$ is a compact subset of **R** with usual metric.

2. (a) Show that the function 3

$$d(x, y) = \frac{|x-y|}{1+|x-y|}$$ is a metric on $\mathbf{R}$.

(b) Give an example of a sequence of functions such that the inequality in the Fatou's lemma should be a strict inequality. 3

(c) Find the extreme values of the function $f(x, y, z) = xyz$ subject to the constraint 4

$$\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25} = 1, \; x, y, z \geqslant 0$$

3. (a) Let A be a non empty subject of a metric space (X, d). Show that the function $f : X \to \mathbf{R}$ defined by $f(x) = d(x, A)$ is uniformly continuous on $\mathbf{R}$. 4

(b) Check whether the following function is measurable 3

$$f(x) = x \sin \frac{1}{x} \qquad \forall x \neq 0$$

$$f(0) = 0$$

(c) Find the interior, closure and boundary of the set 3

$S\{(x, y) \in \mathbf{R}^2 : x = 0\}$ in $\mathbf{R}^2$

4. (a) Let $f : \mathbf{R}^2 \to \mathbf{R}$ be defined by : 5

$$f(x, y) = \begin{cases} \dfrac{xy\left(x^2 + y^2\right)}{x^2 - y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0 & , \text{if } (x, y) = (0, 0) \end{cases}$$

Check whether the mixed second order partial derivatives exist at (0, 0). Does the second order derivative of this function exist at (0, 0). Justify your answer.

(b) Let $\{f_n\}$ be a sequence of measurable functions on **R** such that $\sum_{n=1}^{\infty} \int |f_n| \, dm < \infty$. **5**

Show that the series $\sum_{n=1}^{\infty} f_n(x)$ converges for almost all x and its sum is integrable.

Further show that :

$$\int \sum_{n=1}^{\infty} f_n \, dm = \sum_{n=1}^{\infty} \int f_n dm$$

5. (a) State the Inverse function Theorem for vector-valued functions. Show that the functions $f : R^4 \to R^4$ defined by $f(x, y, z, w) = (x + y, x^2 + y^2, wz, yw)$ is locally invertable at the point (0, 1, 1, 1) **4**

(b) Let X, Y be metric spaces. Let $f : X \to Y$ be continuous. Show that f^{-1} (V) is open in X for every open set V in Y **3**

(c) Prove that, every path connected metric space is connected. Is converse true. Justify your answer. **3**

6. (a) Compute the first derivative $f'(a)$ of the function f defined by $f = (\sin x^2, xy)$ at the point $a = (0, 1)$. **5**

(b) Let $f \in L^{'}(\mathbf{R})$. Show that $\hat{f}$, the Fourier transformation of f, is continuous on **R**. Find the Fourier cosine series for the function $f(x) = x^2$, if $0 \leq x \leq \pi$. **5**

7. (a) Let f and g be given by **3**

$$f(t) = \begin{cases} \sqrt{t}, & \text{if } 0 < t < 1 \\ 0, & \text{if } t \leq 0 \text{ or } t \geq 1 \end{cases}$$

$$g(t) = \begin{cases} \sqrt{1-t}, & \text{if } 0 < t < 1 \\ 0, & \text{if } t \leq 0 \text{ or } t \geq 1 \end{cases}$$

Find the convolution f∗g of f and g.

(b) Use the dominated convergence theorem to **3**

find $\lim\limits_{n \to \infty} \int_1^{\infty} f_n(x)\, dx$,

where $f_n(x) = \dfrac{nx}{1 + n^2 x^2}$.

(c) Define and give an example for each of the following concepts in the context of signals and systems. **4**

(i) A reflection system

(ii) A system with memory
