M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M.Sc. (MACS)

00573

Term-End Examination

December, 2013

MMT-002 : LINEAR ALGEBRA

Time : 1½ hours *Maximum Marks : 25*

(Weightage 70%)

Note : *Question No. 5 is* ***compulsory****. Answer* ***any three*** *questions from question* ***No. 1*** *to* ***4****. Use of calculators is* ***not*** *allowed.*

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1. (a) Let $\mathbf{P}_3(\mathbf{R})$ be the vector space of polynomials of degree at most 3 and having real coefficients. If $\mathbf{B} = \{1+t,\ t^2,\ t^3-t,\ 3\}$ and $\mathbf{B}' = \{1-t,\ 1+t,\ t^2-t^3,\ t^2+t^3\}$ are ordered bases of $\mathbf{P}_3(\mathbf{R})$ and D is the differential operator on $\mathbf{P}_3(\mathbf{R})$ then find $[D]_{\mathbf{B}}$ and an invertible matrix P such that **3**

$$[D]_{\mathbf{B}'} = P^{-1}[D]_{\mathbf{B}}\,P.$$

(b) Find a QR - decomposition of the matrix **2**

$$\begin{bmatrix} 2 & 1 \\ 0 & -1 \\ 1 & 2 \end{bmatrix}.$$

2. (a) Let A be a linear operator on a finite dimensional vector space over a field F such that the characteristic polynomial of A has all its roots in F. Prove that A is diagonalisable only if for each eigen value λ of A and its algebraic and geometric multiplicities are equal. 2½

(b) Check whether the system $Ax = y$ is consistent, where 2½

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 2 \end{bmatrix} \text{ and } y = \begin{bmatrix} 0 \\ 3 \\ 2 \\ 3 \end{bmatrix}$$

If it is consistent, obtain the solution set. If it is inconsistent, find a least squares solution of the system.

3. (a) Find the Jordan canonical form J of 3

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & -1 \\ 0 & 0 & 2 \end{bmatrix}.$$

(b) Find the spectral decomposition of 2

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

4. Find a singular value decomposition of the matrix **5**

$$\begin{bmatrix} 4 & -15 \\ 3 & 20 \\ 0 & 0 \end{bmatrix}.$$

5. Which of the following statements are true, and which are false ? Justify your answer. **10**

(a) The matrices $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ and $\begin{bmatrix} b & 0 \\ 0 & a \end{bmatrix}$ are unitarily similar.

(b) There is a unique $n \times n$ matrix with spectral radius n.

(c) Every Hermitian matrix is normal.

(d) A matrix is positive definite only if all its entries are positive.

(e) A generalised inverse of a matrix is always invertible.