BME-001

01243

BACHELOR OF TECHNOLOGY IN MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

Term-End Examination

December, 2013

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**. Use of calculator is **allowed**.*

1. Answer **any five** of the following : **5x4=20**

(a) Evaluate the limit of the function

$$f(x)=\frac{|x|}{x}, x \neq 0,$$

if it exists as $x \to 0$.

(b) Examine the differentiability of the function :

$$f(x) = \begin{cases} \frac{x^2}{a} - a & \text{when } x < a \\ 0 & \text{when } x = a \\ a - \frac{a^2}{x} & \text{when } x > a \end{cases}$$

at the point $x = a$.

(c) If $\theta = t^n \, e^{-(r^2/4t)}$, find the value of n which will make

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$$

(d) Show that the function

$$y = \log\,(1+x) - \frac{2x}{1+x}$$

is increasing function of x for all values of x. Deduce that $\log_e\,(1+x) > \frac{2x}{1+x}$ for all values of $x > 0$.

(e) Prove that the curve $\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a, b) for all values of n.

(f) Find the area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$.

(g) Solve the differential equation (Any One) :

(i) $(x+y+1)\ \dfrac{dy}{dx}=1$

(ii) $(1+yx)\ x dy+(1-yx)\ y dx=0$

2. Answer **any four** of the following : **4x4=16**

(a) Determine α if the vectors $\vec{a}=-12\hat{i}+\alpha\hat{k}$, $\vec{b}=3\hat{j}-\hat{k}$ and $\vec{c}=2\hat{i}+\hat{j}-15\hat{k}$ are co-planar.

(b) Find the directional derivative of the function $xy^2+yz^2+zx^2$ along the tangent to the curve $x=t$, $y=t^2$, $z=t^3$ at $(-1, 1, -1)$.

(c) Find div grad r^m.

(d) Show that the vector function

$\vec{F}=(y+z)\hat{i}+(z+x)\hat{j}+(x+y)\hat{k}$ is

solenoidal and find the function $\vec{f}$ such

that $\vec{F}=\text{curl}\ \vec{f}$

(e) Verify Stoke's Theorem for the function; $\vec{F} = x(x\hat{i} + y\hat{j})$ integrated round the square in the plane $z=0$ whose sides are along the lines $x=0$, $y=0$, $x=a$, $y=a$.

(f) Evaluate

$$\int_C \vec{F} \cdot d\vec{r},$$

where $\vec{F} = c\left[-3a\sin^2\theta\cos\theta\,\hat{i} + a(2\sin\theta - 3\sin^3\theta)\,\hat{j} + b\sin 2\theta\,\hat{k}\right]$

and the curve C is given by $r = a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j} + b\theta\,\hat{k}$, θ varying from $\pi/4$ to $\pi/2$

(g) If $\vec{f} = (x+y+1)\hat{i} + \hat{j} + (-x-y)\hat{k}$ prove that $\vec{f}$ curl $\vec{f} = 0$

(h) Show that $\frac{1}{3}\int_s \vec{r} \cdot d\vec{a} = V$, where V is the volume enclosed by the surfaces S

3. Answer **any six** of the following : **6x3=18**

(a) (i) Determine the dimension of the subspace $\{(x_1, x_2, x_3, x_4) \mid x_1 = x_2 \text{ and } x_3 = x_4\}$ of R^4.

(ii) Show that $\{(1, -1, 0), (0, 1, -1)\}$ is a basis of the subspace $\{(x, y, z) \mid x + y + z = 0\}$ of R^3.

(b) For what values of λ, the following equations are consistent :

$$ax + hy + g = 0$$

$$hx + by + f = 0$$

$$gx + fy + (c - \lambda) = 0.$$

(c) Verify that the matrix $\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is orthogonal.

(d) If A is a 3×3 matrix, $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 4 \\ -1 & 4 & 3 \end{bmatrix}$

show that $A + A'$ is symmetric and $A - A'$ is show symmetric.

(e) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 0 & -1 \end{bmatrix},$$

using Cayley - Hamilton Theorem.

(f) Find the rank of the matrix $\begin{bmatrix} 4 & 3 & -5 \\ 1 & 1 & 0 \\ 1 & 1 & 5 \end{bmatrix}$,

using elementary row - operations.

(g) Find the eigen-vectors for the matrix

$$A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}.$$

(h) Solve the linear system of equations

$$x - y + z = 3$$

$$3x + 2z = 7$$

$$x - 4y + 2z = 5,$$

if consistent, using matrices.

4. Answer **any four** of the following : **4x4=16**

(a) A company has two plants to manufacture cars. Plant I manufactures 80% of the cars and Plant II manufactures 20%. At plant I, 85 out of 100 cars are rated standard quality. At plant II, only 65 out of 100 cars are rated standard. What is the probability that cars selected at random came form plant I when it is known that the car is of standard quality.

(b) There are 64 beds in a garden and 3 seeds of a particular type of flower are sown in each bed. The probability of flower being white is 1/4. Find the number of beds with 3 and 0 flowers.

(c) In a certain manufacturing process, 5% of tools produced turn out to be defective. In a sample of 40 tools, find the probability that utmost 2 tools will be defective (Given $e^{-2} = 0.135$).

(d) The marks obtained in an examination follow normal distribution with mean 45 and standard deviation 10. If 1000 students appeared at the examination, calculate the number of students scoring more than 60 marks.

(e) A sample of size 9 from a normal population gave $\bar{x} = 15.8$ and $S_x^2 = 10.3$. Find a 99% interval for population mean ($t_{0.01}$ for 8 d.f. $= 3.36$).