

B.Tech. Civil (Construction Management) /
B.Tech. Civil (Water Resources Engineering) /
B.Tech. (Aerospace Engineering)

Term-End Examination
December, 2013

ET-102 : MATHEMATICS - III

Time : 3 hours

Maximum Marks : 70

Note : Answer *any ten* questions . Use of calculator is *allowed*.

1. Test the series $x + \frac{x^3}{3} + \frac{x^5}{5} + \dots$ for convergence for all positive values of x . 7
2. Using Cauchy's Integral test, show that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ converges if $p > 1$ and diverges if $0 < p \leq 1$. 7
3. Find half-range sine series for the function $f(x) = x(\pi - x)$ for $0 \leq x \leq \pi$ 7
4. Show that the series $1 + x + \frac{x^2}{\sqrt{2}} + \dots$ converges absolutely for all values of x . 7
5. Apply the Hurwitz-Routh criterion to determine the stability of the system whose characteristic equation is given by $s^4 + 5s^2 + s + 10 = 0$. 7

6. Find the characteristic function, transfer function, frequency response function and characteristic roots of the equation $(D^3 + D^2 + D + 1)y = e^{2x}$ Also find its particular solution. 7
7. (a) Evaluate $L[e^{-2t}(3 \cos 6t + 5 \sin 6t)]$, 3+4
where L denotes Laplace Transform.
- (b) Find $L^{-1}\left[\frac{e^{-2s}}{(s-2)^4}\right]$.
8. Solve the differential equation $y'' + 2y' + 5y = e^{-t}\sin t$ given that $y(0) = 0$, $y'(0) = 1$, using Laplace transform method. 7
9. Find the temperature $u(x, t)$ in a bar of length x which is perfectly isolated, also at the ends $x=0$ and $x=\pi$, assuming that $c=1$ and $u(x, 0) = x$. (Formulate the problem and use method of separation of variables for finding the solution) 7
10. Solve the PDE 7
 $(D^2 - DD' - 2D'^2 + 2D + 2D')Z = \sin(2x+y)$.
11. Find the solution near $x=0$ of the differential equation $9x(1-x)y'' - 12y' + 4y = 0$. 7
12. (a) Solve $x \log x \frac{dy}{dx} + y = 2 \log x$ 4+3
(b) Find the particular integral of $(D^2 - 4D + 4)y = x^2 + \cos 2x$.

13. Find the orthogonal trajectories of the family of circle $x^2 + (y - c)^2 = c^2$, where c is a parameter. 7

14. Show that the function $v = -\frac{y}{(x^2 + y^2)}$ is harmonic 7
and find its conjugate.

15. Find the bilinear transformation whose fixed points are 2 and 3. 7

16. Using Complex Variables, prove that 7

$$\int_0^{\pi} \frac{3d\theta}{9 + \sin^2\theta} = \frac{\pi}{\sqrt{10}}$$

17. Determine the poles of the function 7

$$f(z) = \frac{3z+2}{z^3 - 6z^2 + 11z - 6} \text{ and the residue at each}$$

pole Hence evaluate $\int_{c: |z-i|=2} f(z)$
