ET-102

B.Tech. Civil (Construction Management) /
B.Tech. Civil (Water Resources Engineering)

Term-End Examination

June, 2008

ET-102 : MATHEMATICS III

Time : 3 hours *Maximum Marks : 70*

Note : *Answer any* ***ten*** *questions. All questions carry equal marks. Use of calculator is allowed.*

1. Test the convergence or divergence of the series

$$\sum_{n=1}^{\infty} \left[\sqrt{n^2+1}-n\right]$$

2. Test for convergence the following series :

$$1+\frac{1^2.2^2}{1.3.5}+\frac{1^2.2^2.3^2}{1.3.5.7.9}+ \dots \infty$$

3. Discuss the convergence of the series

$$x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\frac{5^5x^5}{5!}+ \dots \infty$$

4. Find the Fourier series expansion for f(x), if

$$f(x) = -\pi, \quad -\pi < x < 0$$
$$ = x, \quad 0 < x < \pi.$$

Deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}.$$

5. Find the half-range sine series for the function

$$f(t) = t - t^2, \quad 0 < t < 1.$$

6. If $2 \cos \theta = x + \frac{1}{x}$, prove that $2 \cos r\theta = x^r + \frac{1}{x^r}$.

7. Show that the polar form of Cauchy – Riemann equations are $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$, $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$.

Deduce that

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

8. Determine p such that the function

$$f(z) = \frac{1}{2} \ln (x^2 + y^2) + i \tan^{-1}\left(\frac{px}{y}\right)$$

be an analytic function.

9. Find the bilinear transformation which maps the points $-1, i, 1$ of the z-plane on $1, i, -1$ of the w-plane respectively.

10. Evaluate $\int_{1-i}^{2+3i} (z^2 + z)\, dz$ along the line joining the points (1, –1) and (2, 3).

11. Find the Laplace transform of f(t) defined as

$$f(t) = t/\tau \quad \text{when} \quad 0 < t < \tau$$
$$= 1 \quad \text{when} \quad t > \tau$$

12. Find the inverse Laplace transforms of $\frac{s+2}{s^2(s+1)(s-2)}$.

13. Solve the differential equation by the Laplace transform method :

$$y'' + y = t, \quad y(0) = 1, \quad y'(0) = -2.$$

14. The differential equation for a circuit in which self-inductance and capacitance neutralise each other is

$$L\frac{d^2i}{dt^2} + \frac{i}{c} = 0.$$

Find the current i as a function of t given that I is the maximum current, and i = 0 when t = 0.

15. Solve :

$$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}.$$