MTE-11

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

PROBABILITY AND STATISTICS

(Valid from 1st January, 2021 to 31st December, 2021)

It is compulsory to submit the assignment before filling the exam form.

School of Sciences
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(2021)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………..…

STUDY CENTRE: ………………………..….. **DATE:** ……………………….……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2021**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.
We wish you good luck.

ASSIGNMENT

Course Code: MTE-11
Assignment Code: MTE-11/TMA/2021
Maximum Marks: 100

1. State whether the following statements are true or false. Give a short proof or a counter example in support of your answers: (10)

   (a) Poisson distribution is a limiting case of binomial distribution for $\mathbf{n \to \infty, p \to 1}$ and $np \to \infty$.

   (b) For two independent events A and B, if $P(A) = 0.2$ and $P(B) = 0.4$, then $(A \cap B) = 0.6$.

   (c) If $H_0 : P \leq 0.6$ and $X \sim B(n, p)\, n$-known and p unknown and $H_1 : \mu = \mu_0$ where $X \sim N\ (\mu, \sigma^2)\sigma^2$ unknown, then H_0 and H_1 are simple null hypothesis.

   (d) Frequency density of a class for any distribution is the ration of total frequency to class width.

   (e) If X and Y are independent r.v.s with $M_X(t)$ and $M_Y(t)$ as their m.gf's respectively, then $M_{X+Y}(t) = M_X(t) M_Y(t)$.

2) A, B and C are three events. Express the following events in set notations. (10)

   (i) Simultaneous occurrence of A, B and C.
   (ii) Occurrence of at least one of them.
   (iii) Both A and B occur and C does not occur.
   (iv) The event B but not A occurs.
   (v) Not more than one of the events A, B and C occur.

3) (a) If the moment generating function (m.g.f.) of a random variable X is $M_X(t) = \exp(3t + 32t^2)$. Find mean and standard derivation of X and also compute $P(x < 3)$. (6)

   (b) The probability density function of a random variable X is $f(x) = C\,|\,x\,|$; Find C, and the value of x_0 such that $F_X(x_0) = \dfrac{3}{4}$, where F is the CDF. (4)

4. (a) Five unbiased dice were thrown 96 times and the number of times 4, 5 or 6 was obtained, is given in the following table: (5)

No. of dice showing 4, 5 or 6	0	1	2	3	4	5
Frequency	1	10	24	35	18	8

At 5% level of significance test whether this data comes from a binomial distribution. You may like to use the following values.

$[(x_5^2 10.05) = 11.07, \quad x_6^2 (0.05) = 12.59, \quad x_7^2 (0.05) = 14.07.]$

(b) The yield (in kg) of 100 plots in the form of grouped frequency distribution is given below: (5)

Yield (kg)	**Frequency**
0-20	6
20-40	21
40-60	35
60-80	30
80-100	8

(i) Estimate the number of plots with an yield of

(A) 40 to 80 kg

(B) 10 to 70 kg

(ii) Find the mean and standard deviation of yield.

5. (a) Suppose X is a gamma variate with $E(x) = 3$ and $\text{var}(X) = 7$. Find the parameters α and λ of the gamma distribution. (5)

(b) For the given bivariate probability distribution of X and Y: (5)

$$P(X = x, Y = y) = \frac{x^2 + y}{32} \text{ for } x = 0,1,2,3 \text{ and } y = 0,1.$$

Find:

(i) $P(X \leq 1, Y = 1)$

(ii) $P(X \leq 1)$

(iii) $P(Y > 0)$ and

(iv) $P(Y = 1 \mid X = 3)$

6. (a) For normal distribution with mean zero and variance σ^2 show that: (5)

$$E(|x|) = \sqrt{\frac{2}{\pi}}\sigma.$$

(b) If a random variable u has t-distribution with n degree of freedom, find the distribution of u^2. (5)

7. (a) A factory produces steel pipes in three plant with daily production volumes of 500, 1000 and 2000 units respectively from each of the plants. From the past experience it is known that the fraction of defective outputs produced by three plants are respectively 0.005, 0.008 and 0.010. If a pipe is selected at random from a day's total production and founded to be defective, from which plant is that likely to have came? (5)

(b) Let $X_1, X_2, \ldots, X_n$ be random sample of size n from a distribution with probability density function $f(X;,0) = \begin{cases} \theta X^{\theta-1}, 0 < X < 1, \theta > 0 \\ 0, \text{ else where.} \end{cases}$ Obtain a maximum likeyhood estimator of θ. (5)

8. (a) Let $X_1, X_2, \ldots, X_n$ be independently and identically distributed $b(1, p)$ random variables. Obtain a confidence internal for p using Chebychev's inequality. (5)

(b) For 25 army personnels, line of regression of weight of kidneys (Y) on weight of heart (X) is $Y = 0.399X + 6.934$ and the line of regression of weight of heart on weight of kidney is $X - 1.212Y + 2.461 = 0$. Find the correlation coefficient between X and Y and their mean values. (5)

9. (a) Let X be a binomial variate with $n = 100, p = 0.1$. Find the approximate value of $P(10 \leq X \leq 12)$ using: (5)

(i) normal distribution

(ii)poisson distribution

[You may like to use the following values.

$P(Z \leq 0.67) = 0.7486$, $P(Z \leq 0.33) = 0.6293$, $P(Z \leq 0) = 0.5$]

(b) For the given distribution: (5)

$P(X = x) = \frac{2}{3}\left(\frac{1}{3}\right)^x$; $x = 0, 1, 2, \ldots,$ find moment generating function, mean and variance of X.

10. (a) For a distribution, the mean is 10, variance is 16, the skewness sk_4 is $+1$ and kurtosis b_2 is 4. Obtain the first four moments about the origin i.e. zero. Comment upon the nature of the distribution. (5)

(b) Find the mean and variance of binomial distribution. (5)