MTE-08

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

DIFFERENTIAL EQUATIONS

Valid from 1st January, 2021 to 31st December, 2021

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2018)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE : …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE : …………………………..….. DATE :……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2021. If you have failed in this assignment or fail to submit it by December, 2021, then you need to get the assignment for the year 2022 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MTE-08
Assignment Code: MTE-08/TMA/2021
Maximum Marks: 100

1. Classify the following as true or false giving reasons for your answers..

   i) For the IVP, $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$, the continuity of $f(x, y)$ and $\frac{\partial f}{\partial y}$ guarantees the unique solution of the problem.

   ii) Equations $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + x^2 y = e^{x^2/2}$ and $\frac{d^2y}{dx^2} + y = 1$ have the same normal form.

   iii) Equation $\cos(x+y)p + \sin(x+y)q = z^2 + z$ is a quasi-linear equation.

   iv) $\frac{\partial^2 z}{\partial x^2}\frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x \partial y}\right)^2 = 0$ is a non-linear PDE.

   v) Every solution of the ordinary differential equation $(D^2+1)^2 y = 0$ is bounded on $[0, \infty[$. (10)

2. a) Find the differential equation of the family of curves $x^2 + y^2 + 2ax + 2by + c = 0$, where a, b, c are parameters. (2)

   b) Do the functions $y_1(t) = \sqrt{t}$ and $y_2(t) = \frac{1}{t}$ form a fundamental set of solutions of the equation $2t^2 y'' + 3t\,y' - y = 0$, on the interval $0 < t < \infty$? Justify your answer. (2)

   c) A solution of the IVP

   $$(1-t^2)\frac{d^2y}{dt^2} + 2t\frac{dy}{dt} - 2y = 0,\ y(0) = 3,\ y'(0) = -4$$

   is $y_1 = t$. Use the method of reduction of order to find a general solution of IVP on the interval $-1 < t < 1$. (3)

   d) If $\frac{d^2x}{dt^2} + \frac{g}{b}(x-a) = 0$, ($a, b, g$ being positive constants) and $x = a'$ and $\frac{dx}{dt} = 0$ when $t = 0$, show that $x = a + (a'-a)\cos\left\{\frac{\sqrt{g}}{b}t\right\}$. (3)

3. Solve the following differential equations

   a) $\sin^{-1}(dy/dx) = x + y$ (3)

   b) $(1+y^2)dx = (\tan^{-1} y - x)dy$ (4)

   c) $(D-1)^2(D^2+1)^2 y = \sin^2\left(\frac{x}{2}\right) + e^x + x$ (4)

d) $2x^2y\left(\frac{d^2y}{dx^2}\right)+4y^2=x^2\left(\frac{dy}{dx}\right)^2+2xy\left(\frac{dy}{dx}\right)$ (4)

4. a) Suppose the temperature of a body when discovered is 85° F. Two hours later, the temperature is 74° F and the room temperature is 68° F. Find the time when the body was discovered after death (assume the body temperature to be 98.6° F at the time of death.) (4)

b) Find a continuous solution of the IVP

$$\frac{dy}{dx}+y=g(t),\ y(0)=0$$

where $g(t)=\begin{cases}2, & 0\le t\le 1\\ 0, & t>1\end{cases}$. (3)

c) Solve the differential equation

$$x\cos\left(\frac{y}{x}\right)(y\,dx+x\,dy)=y\sin\left(\frac{y}{x}\right)(x\,dy-y\,dx).$$ (3)

5. a) A certain population is known to be growing at a rate given by the logistic equation

$$\frac{dx}{dt}=x(b-ax)$$

Show that the minimum rate of growth will occur when the population is equal to half the equilibrium size, that is, when the population is $b/2a$. (5)

b) Identify the following differential equations and hence solve them

i) $y'=-\frac{4}{x^2}-\frac{y}{x}+y^2$ (3)

ii) $y=xy'+1-\ln y'$. (4)

c) Using the method of undetermined coefficients, find the general solution of the DE

$$y^{iv}-2y'''+2y''=3e^{-x}+2e^{-x}x+e^{-x}\sin x.$$ (3)

6. a) Verify that the equations

i) $z=\sqrt{2x+a}+\sqrt{2y+b}$ and

ii) $z^2+\mu=2(1+\lambda^{-1})(x+\lambda y)$

are both complete integrals of the PDE $z=\frac{1}{p}+\frac{1}{q}$. Also show that the complete integral (ii) is the envelope of the one parameter sub-system obtained by taking $b=-\frac{a}{\lambda}-\frac{\mu}{1+\lambda}$ in the solution i). (6)

b) Find the differential equations of the space curve in which the two families of surfaces $u=x^2-y^2=c_1$ and $v=y^2-z^2=c_2$ intersect. (2)

c) Find the value of n for which the equation $(n-1)^2u_{xx}-y^{2n}u_{yy}=ny^{2n-1}u_y$ is parabolic or hyperbolic. (2)

7. a) Find the general integral of the equation

$$(x-y)p+(y-x-z)q=z$$

and the particular solution through the circle $z=1,\ x^2+y^2=1$. (5)

b) Find the surface which is orthogonal to the one parameter system $z = c\,xy(x^2 + y^2)$ and which passes through the hyperbola $x^2 - y^2 = a^2,\ z = 0$. (5)

8. a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. It is released from rest from the initial position. Find the displacement $y(x,t)$. (7)

b) Find the integral surface of the PDE

$$(x - y)p + (y - x - z)q = z\ .$$ (3)

9. a) Find the complete integral of the equation

$$\left(\frac{\partial z}{\partial x_1}\right)\left(\frac{\partial z}{\partial x_2}\right)\left(\frac{\partial z}{\partial x_3}\right) = z^3 x_1 x_2 x_3\ .$$ (5)

b) Solve the following PDEs:

i) $(3D^2 - 2D^2 + D - 1)z = 4e^{x+y}\cos(x + y)$

ii) $(D + D' - 1)\ (D + 2D' - 3)z = 4 + 3x + 6y$. (5)