MTE-02

ASSIGNMENT BOOKLET

(Valid from 1st January, 2021 to 31st December, 2021)

Bachelor's Degree Programme

Linear Algebra

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi
(2021)

Dear Student,
Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 percent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

	ROLL NO. : ....
	NAME : ....
	ADDRESS : ....
	....
	....
COURSE CODE :	....
COURSE TITLE :	
STUDY CENTRE :	**DATE** ....

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **Solve the assignment on your own. Don't copy from your fellow students or from the internet. If you are found guilty of copying, action will be taken for using unfair means and you will not be allowed to appear for the Term End Examination.**

7) This assignment is to be submitted to the Programme Centre as per the schedule made by the Programme Centre. Answer sheets received after the due date shall not be accepted.

8) This assignment is valid only up to December, 2020. If you fail in this assignment or fail to submit it by December, 2021, then you need to get the fresh assignment for the year 2022 and submit it as per the instructions given in the Programme Guide.

9) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So, solve it and **submit it to your study centre at the earliest.**

10) We **strongly** suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment

Course Code: MTE-02
Assignment Code: MTE-02/TMA/2021
Maximum Marks: 100

1) a) Which of the following are binary operations on $\mathbf{R}$? Justify your answer.

   i) The operation $\bigtriangledown$ defined by $x \bigtriangledown y = x \sin y$.

   ii) The operation $\bigtriangleup$ defined by $x \bigtriangleup y = e^{x-y}$.

   Also, for those operations which are binary operations, check whether they are associative and commutative. (5)

   b) Find the vector equation of the plane determined by the points $(1,0,-1)$, $(0,1,1)$ and $(-1,1,0)$. Also find the point of intersection of the line $\mathbf{r} = (1+t)\mathbf{i} + (1-2t)\mathbf{j} + (2+t)\mathbf{k}$ and the plane . (3)

   c) Let $\mathbf{u} = \frac{2\mathbf{i}+2\mathbf{j}+\mathbf{k}}{3}$, $\mathbf{v} = \frac{\mathbf{i}-\mathbf{j}}{\sqrt{2}}$ and $\mathbf{w} = \frac{-\sqrt{2}(\mathbf{i}+\mathbf{j}-4\mathbf{k})}{6}$. Compute the scalar products $\mathbf{u}\cdot\mathbf{v}$, $\mathbf{u}\cdot\mathbf{w}$ and $\mathbf{v}\cdot\mathbf{w}$. Check whether $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are orthonormal. (2)

2) a) Which of the following are subspaces of $\mathbf{R}^3$? Justify your answer.

   i) $S = \{(x,y,z) \in \mathbf{R}^3 \,|\, x+z = 2y\}$

   ii) $S = \left\{(x,y,z) \in \mathbf{R}^3 \,\middle|\, x+yz = 0\right\}$

   For those subsets which are subspaces, find a basis. (5)

   b) Check that $B = \{1, 2x+1, (x-1)^2\}$ is a basis for $\mathbf{P}_2$. Find the coordinates of $1-x-2x^2$ with respect to this basis. (5)

3) Let $T : \mathbf{R}^3 \to \mathbf{R}^4$ be defined by

$$T(x_1,x_2,x_3) = (x_1+2x_3, x_2-x_3, x_1-x_3, 2x_1+x_2-x_3)$$

Check that T is a linear operator. Find the kernel and range of T. Find the dimension of the kernel. (10)

4) a) For the vector space $\mathbf{P}^2$, find the dual basis of $\{1+x, 1+2x, 1+x+x^2\}$. (3)

   b) Define $T\colon \mathbf{R}^3 \to \mathbf{R}^3$ by

$$T(x,y,x) = (x, x-y, x+2y-z).$$

   Check whether T satisfies the polynomial $(x-1)(x+1)^2$. (3)

   c) Let $T\colon \mathbf{R}^3 \to \mathbf{R}^3$ be a linear operator and suppose the matrix of the operator with respect to the ordered basis

$$B = \left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$$

   is $\begin{bmatrix}1 & 0 & -1\\0 & 1 & 1\\1 & 0 & 1\end{bmatrix}$. Find the matrix of the linear transformation with respect to the basis

$$B' = \left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\0\\-1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$$ (4)

5) a) Consider the following system of equations: (6)

$$x + 2z + w = 1$$
$$y + z + 2w = 1$$
$$x - 2y - 3w = 2$$

i) Write the system of equations in the form of a matrix equation $AX = B$.

ii) Find the rank of the matrix A and rank of the augmented matrix $[AB]$. Hence determine whether the system of equations has a solution or not. (6)

b) Let $T\colon \mathbf{P}_2 \to \mathbf{P}_2$ be defined by

$$T(a+bx+cx^2) = b - 2cx + 2(a-b)x^2.$$

Check that T is a linear transformation. Find the matrix of the transformation with respect to the ordered bases $B_1 = \{x^2, x^2-x, x^2-x-1\}$ and $B_2 = \{1, x, x^2\}$. Find the kernel of T. (4)

6) a) Check whether the matrices A and B are diagonalisable. Diagonalise those matrices which are diagonalisable. (11)

i) $A = \begin{bmatrix} 0 & -3 & 1 \\ -1 & -2 & 1 \\ -2 & -6 & 3 \end{bmatrix}$ ii) $B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 3 & 0 \\ 1 & -1 & 1 \end{bmatrix}$.

b) Find inverse of the matrix B in part a) of the question by using Cayley-Hamiltion theorem if it exists. Otherwise, find its minimal polynomial. (2)

c) Find the inverse of the matrix A in part a) of the question by finding its adjoint if the inverse exists. Otherwise find its minimal polynomial (3)

7) a) Let $\mathbf{P}_3$ be the inner product space of polynomials of degree at most 3 over $\mathbf{R}$ with respect to the inner product

$$\langle f, g\rangle = \int_0^1 f(x)g(x)\,dx.$$

Apply the Gram-Schmidt orthogonalisation process to find an orthonormal basis for the subspace of $\mathbf{P}_3$ generated by the vectors (8)

$$\left\{1+x, 1-x^2, 1-x^3\right\}.$$

b) Consider the linear operator $T\colon \mathbf{C}^3 \to \mathbf{C}^3$, defined by

$$T\left(z_1, z_2, z_3\right) = \left(z_1 - iz_2, iz_1 - 2z_2 + iz_3, -iz_2 + z_3\right).$$

i) Compute T^* and check whether T is self-adjoint.

ii) Check whether T is unitary. (6)

c) Let $\left(x_1, x_2, x_3\right)$ and $\left(y_1, y_2, y_3\right)$ represent the coordinates with respect to the bases $B_1 = \{(1,0,0),(1,1,0),(0,0,1)\}$, $B_2 = \{(1,0,0),(0,1,1),(0,0,1)\}$. If $Q(X) = x_1^2 - 4x_1x_2 + 2x_2x_3 + x_2^2 + x_3^2$, find the representation of Q in terms of $\left(y_1, y_2, y_3\right)$.(3)

d) Find the orthogonal canonical reduction of the quadratic form $-x^2 + y^2 + z^2 + 4xy + 4xz$. Also, find its principal axes. (7)

8) Which of the following statements are true and which are false? Justify your answer with a short proof or a counterexample. (10)

i) The Row Reduced Echelon form of any invertible square matrix is the identity matrix.

ii) If W_1 and W_2 are proper subspaces of a non-zero, finite dimensional, vector space V and $\dim(W_1) > \frac{\dim(V)}{2}$, $\dim(W_2) > \frac{\dim(V)}{2}$, the $W_1 \cap W_2 \neq \{0\}$.

iii) If V is a vector space and $S = \{v_1, v_2, \ldots, v_n\} \subset V$, $n \geq 3$, is such that $v_i \neq v_j$ if $i \neq j$, then S is a linearly independent set.

iv) If $T_1, T_2 : V \to V$ are linear operators on a finite dimensional vector space V and $T_1 \circ T_2$ is invertible, $T_2 \circ T_1$ is also invertible.

v) If $T_1, T_2 : V \to V$ are self adjoint operators on a finite dimensional inner product space V, then $T_1 + T_2$ is also a self adjoint operator.