MTE-01

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

CALCULUS

(Valid from 1st January, 2021 to 31st December, 2021)

It is compulsory to submit the assignment before filling the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068

(2021)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

……………………………………………

……………………………………………

COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………….

STUDY CENTRE: ……………………………. **DATE:** ……………………………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2021**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

Course Code: MTE-01
Assignment Code: MTE-01/TMA/2021
Maximum Marks: 100

1. Which of the following statements are true. Give a short proof or a counter example is support of your answer. (10)

   (a) The function f, given by

   $f(x) = \frac{1}{6}(x^3 - 6x^2 + 9x + 6)$, has a point of inflection.

   (b) $\frac{d}{dx}\left[\int_3^{3x^2} \tan t^2 \, dt\right] = 6x\sec^2(3x^2)$.

   (c) The function $y = \sin x$ is monotonic on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

   (d) The graph of the function $y = x - |x|$ lies in the 3rd quadrant only.

   (e) The tangent to the curve $x^2 + y^2 - 2x = 0$ at the point $(2,0)$ is parallel to the x-axis.

2) (a) If $y = e^{m \tan^{-1} x}$, show that (5)

   $(1+x^2)\, y_{n+1} + (2nx - m)\, y_n + n(n-1)\, y_{n-1} = 0.$

   (b) Write down the Taylor's series for $\cos 4x$ around zero. Hence, find out for which (5) value(s) of k the function f, given by

   $$f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & \text{when } x \neq 0 \\ k(2+\sin^2 x), & \text{when } x = 0 \end{cases}$$

   is continuous at $x = 0$.

3) (a) Find the length of the curve given by $x = e^t \cos t, y = e^t \sin t$ lying in $0 \le t \le \pi$. (5)

   (b) Find the derivative of $\cos^{-1}(1-2x^2)$ with respect to $\cos^{-1}(\sqrt{1-x^2})$. (5)

4. (a) Evaluate $\int \frac{x+3}{\sqrt{x^2+4x+5}} dx.$ (5)

   (b) Give an example of a function which is one-one when defined on a domain $D_1 \subseteq \mathbb{R}$, but not when defined on a domain $D_2 \subseteq \mathbb{R}$. Justify your choice of example. (3)

(c) Give an example, with justification, of a function with domain $[2, 5]$ which is **not** integrable. (2)

5. (a) Find the maximum height of the curve $y = 4\sin^2 x - 3\cos^2 x$ above the x-axis. (5)

(b) Evaluate $\int \frac{(4-2x)\,dx}{(x^2+1)(x-1)^2}$. (5)

6. (a) Find the intervals of $\mathbb{R}$, where the function f, defined by $f(x) = x^3 - 27x + 36$, is increasing or decreasing. (5)

(b) Prove that (5)

$$I_n = \int_{\pi/4}^{\pi/2} \cot^n x\,dx = \frac{1}{n-1} - I_{n-2}, \text{ and hence evaluate } I_4.$$

7. (a) Find the equations of the tangent and normal to the curve (5)

$x = t^2, y = t^3$ at $t = 2$.

(b) Find an approximate value of $\ln 2$, by solving the definite integral $\int_1^2 \frac{dx}{x}$, using the Trapezoidal rule with 5 ordinates. (5)

8. Trace the curve $y = x + \frac{1}{x}$, stating all the properties you use for doing so. (10)

9. (a) Find the area of the region bounded by the curve $a^4 y^2 = x^5(2a - x)$. (5)

(b) Graph the function f, defined by $f(x) = |x| + |x-1|$. Also, give its domain and range. (5)

10. (a) Evaluate $\int_0^2 [x]\,dx$. (4)

(b) Find the derivative of $x^{\tan x} + (\sin x)^{\cos x}$ w.r.t.x. (6)