MTE-09

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(B.Sc./B.A./B.Com.)

REAL ANALYSIS

Valid from 1st January 2022 to 31st December 2022

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

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THE PEOPLE'S UNIVERSITY

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2022)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:…………………………………………

NAME :…………………………………………

ADDRESS :…………………………………………

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…………………………………………

COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: ………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE FORMAT ABOVE STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of the very thin variety) for writing your answers.
3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2022. If you have failed in this assignment or fail to submit it by December, 2022, then you need to get the assignment for the year 2023 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

ASSIGNMENT

Course Code: MTE-09
Assignment Code: MTE-09/TMA/2022
Maximum Marks: 100

1. Are the following statements true or false? Give reasons for your answers. (10)

   a) 2 is a limit point of the interval $[1,4]$.

   b) Every bounded sequence is convergent.

   c) The function, $f:\mathbf{R}\to\mathbf{R}$ defined by $f(x)=|x-1|+|3-x|$ is differentiable at $x=4$.

   d) The function, $f(x)=[x]-x$, is not integrable in $[0,3]$, where $[x]$ denotes the greatest integer function.

   e) The function f defined by

   $$f(x)=\begin{cases}\dfrac{e^x+e^{-x}}{2}, & \text{when } x\neq 0\\ \dfrac{1}{2}, & \text{when } x=0\end{cases}$$

   is continuous in the closed interval, $[-1,1]$.

2) a) Find $\lim\limits_{x\to 0}\dfrac{\tan x\sec^2 x - x}{x^3}$ (3)

   b) Examine whether the equation, $x^3-11x+9=0$ has a real root in the interval, $[-1,2]$. (3)

   c) Check whether the following series are convergent or not (4)

   i) $\sum\limits_{n=1}^{\infty}\dfrac{3n-1}{7^n}$

   ii) $\sum\limits_{n=1}^{\infty}\dfrac{\sqrt{n^2+3}-\sqrt{n^2-3}}{\sqrt{n}}$

3) a) Does the sequence $(3+(-1)^n)$ converge to 2? Justify. (2)

   b) Show that $\lim\limits_{x\to\infty}\left(\dfrac{x-3}{x+1}\right)^x=e^{-4}$. (3)

   c) Check whether the sequence (f_n) where $f_n(x)=\dfrac{3x}{1+nx^2}, x\in[2,\infty[$ is uniformly convergent in $[2,\infty[$. (5)

4. a) Using principle of induction, prove that 64 is a factor of $3^{2n+2} - 8n - 9, \forall n \in \mathbf{N}$. (3)

b) Find $\lim_{n\to\infty}\left[\frac{1}{(2n+1)^2} + \frac{2}{(2n+2)^2} + \frac{3}{(2n+3)^2} + ... + \frac{3}{25n}\right]$ (3)

c) Show that the function f defined by $f(x) = \frac{1}{x^2}$ is uniformly continuous on $[5, \infty]$. (4)

5. a) Test the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3}{5n-2}$ for absolute and conditional convergence. (4)

b) Find the local maximum and the local minimum values of the function $f(x) = x^3 - 2x^2 - 4x + 5$. (4)

c) Examine the following series for convergence:

$$\sum_{n=1}^{\infty}\left(\frac{n-2}{2n+3}\right)^n$$ (2)

6. a) Show that the function f defined on $[0,1]$ by $f(x) = (-1)^{n-1}$ for $\frac{1}{n+1} < x \le \frac{1}{n}$ $(n = 1, 2, 3, ...)$ is integrable on $[0,1]$. (3)

b) For the function f defined by $f(x) = 3x - 2$, over $[0,1]$, verify $L(P, f) \le U(P, f)$ where the partition P is $\left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$. (4)

c) Prove that $\lim_{x\to -5} \frac{1}{(x+5)^2} = \infty$, with proper justification. (3)

7. a) Show that $R_n(x)$, the Lagrange's form of remainder in the Maclaurin series expansion of $\cos 3x$ tends to zero as $n \to \infty$. Hence obtain its infinite Maclaurin expansion. (5)

b) Consider the function, f defined by $f(x) = |x-3| + [x], x \in [2, 4]$ where $[x]$ denotes the greatest integer function. Is this function differentiable in $[2, 4]$? Justify your answer. (3)

c) Let a function $f : \mathbf{R} \to \mathbf{R}$ be defined by

$$f(x) = \begin{cases} 2, & \text{if } x \in Q \\ 4, & \text{if } x \notin Q \end{cases}$$

Show that f is not continuous at any $x \in \mathbf{R}$. (2)

8. a) Prove or disprove the following statement: (2)

'Every strictly increasing onto function is invertible'.

b) Examine the continuity of the function $f:[1,3] \to \mathbf{R}$ defined by

$$f(x) = \frac{[x]}{3x-2}$$

where $[x]$ denotes the greatest integer function. (4)

c) Show that $L(P_2, f) \le U(P_1, f)$ where $f(x) = 3x + 2$ is defined over $[0,1]$ and $p_1 = \left\{0, \frac{1}{2}, \frac{3}{4}, 1\right\}$ and $P_2 = \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}$ (4)

9. a) Find the upper and lower Riemann integrals of the function f, defined on $[a,b]$ as follows:

$$f(x) = \begin{cases} 2, & \text{when } x \text{ is irrational} \\ 0, & \text{when } x \text{ is rational.} \end{cases}$$

Is f Riemann integrable on $[a,b]$? Justify your answer. (5)

b) Show that the function $f(x) = x^3$ is not uniformly continuous on $\mathbf{R}$. Does there exist a subset of **R** such that f restricted to it is uniformly continuous in it? Justify your choice. (5)

10. a) Prove that the series $\sum_{n=1}^{\infty} n^2 x^n$ converges uniformly in the interval $\left[0, \frac{1}{5}\right]$. (3)

b) Show that the sequence $\{f_n\}$ of functions, where $f_n(x) = \frac{n}{x+2n}$, is uniformly convergent in $[0,k]$, where $k > 0$. (4)

c) Examine the function

$$f(x) = 2x^3 - 9x^2 - 60x + 150$$

for extreme values. (3)