MTE-04

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

ELEMENTARY ALGEBRA

(Valid from 1st January, 2022 to 31st December, 2022)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2022)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: ………………………..….. **DATE:** ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **This assignment is valid only upto December, 2022**. If you have failed in this assignment or fail to submit it by the last date, then you need to get the assignment for the next cycle and submit it as per the instructions given in that assignment.

7) It is compulsory to submit the assignment before filling in the exam form.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

ASSIGNMENT

Course Code: MTE-04
Assignment Code: MTE-04/TMA/2022
Maximum Marks: 100

1) Which of the following statements are true? Justify your answers. (This means that if you think a statement is false, give a short proof or an example that shows it is false. If it is true, give a short proof for saying so. (20)

   i) The system of equations $2x+6y+z=2, x+ky+3z=4$ has a unique solution for some value of k.
   ii) Any polynomial equation with real coefficients has at least one real root.
   iii) If A and B are two sets, then $A \cup B = B \cup (A \setminus B)$.
   iv) There is one and only one complex cube root of 1.
   v) The arithmetic mean of 3 non-zero real numbers is greater than their harmonic mean.
   vi) Any pair of linear equations in two variables is consistent.
   vii) If $f(x)=0$ is a polynomial equation of degree n over R, then it must have at least n distinct roots in R.
   viii) A system of linear equations that cannot be solved by Cramer's rule be inconsistent.
   ix) $|x-y| \geq |x|-|y| \; \forall x, y \in R$.
   x) If $z=-1+i$, then $\text{Arg}\left(\frac{1}{z}\right)=\frac{\pi}{4}$.

2) a) Check if the equations can be solved by Cramer's rule. If it can be, then apply the rule for solving the equations; otherwise solve it by the method of Gaussian elimination. (3)

$$-x+3z=2,$$
$$2x+y-4z-=-1$$
$$x+2y+z=4.$$

   b) A concert hall has 400 seats. These seats are divided into two sections A and B. The cost of a ticket in Section A is Rs. 155 and that in Section B is Rs. 105. Assuming that all the seats are occupied, determine the number of seats allocated to each section so as to get a daily revenue of Rs. 50,000/-. (2)

   c) Solve the equation:
   $x^4-10x^3+42x^2-82x+65=0$, given that the product of two of its roots is 13. (5)

3) a) Verify De Morgan's laws for the sets $A=\{5,3,-1\}, B=\{-5,-3,-1\}$. (3)

   b) Can you solve the following system of equations by Cramer's rule? If so, solve it using this rule. If not, solve the given system of equations by elimination method.

$$x+2y+3z=2$$
$$2x+3y=5$$
$$3x+6y+9z=6$$
(2)

   c) Show, by induction, that

$$1^3+2^3+3^3+\cdots+n^3=\left[\frac{n(n+1)}{2}\right]^2, \forall n \geq 1.$$ (3)

d) Find the number of distinct solutions of $2x-3y=7,\ x,\ y<0$ (2)

4) a) For $z_1=3+4i$ and $z_2=4-3i$, write $\frac{z_1}{z_2}$ in polar and exponential form, and represent $\frac{z_1}{z_2}$ and z_1/z_2 in an Argand diagram. (4)

b) For $x>0, n\geq 1$, prove that $1+x+x^2+\cdots+x^{2n}\geq(2n+1)x^n$. (4)

c) If we know that the roots of the cubic equations $x^3-61x^2-8000=0$ are in G.P., then find the roots of the equation. (2)

5) a) Find the roots of the equation

$$g(x)=x^4+7x^3+11x^2+7x+10=0.$$

Given that (x^2+1) divides $g(x)$. (3)

b) Obtain the solution set of the linear system given below, by the Elimination method:

$$x+4y+5z=4$$
$$3x+2y+6z=2$$
$$10y+9z=10$$ (4)

c) Prove that $(n+1)(2n+1)\geq 6(n!)^{2/n}$ for $n\in\mathbf{N}$. (3)

6) a) Prove that, by the principle of mathematical induction,

$$\frac{1}{1.2}+\frac{1}{2.3}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$ for all positive integers n. (4)

b) Akhila enjoyed two types of games, type A and type B, at the village fair. Each time she played type A, it cost ` 3 and each time she played type $B$, it cost ` 4. If the number of B games played was half the number of type A games played by her, and the total amount spent was ` 20, write a system of linear equations for the problem of finding the number of times she played each type of game. (4)

c) Given an example, with justification, of each of the following:

(i) An element of $(\mathbf{C}\times\mathbf{Q})\setminus(\mathbf{Q}\times\mathbf{C})$;

(ii) An infinite set whose complement in $\mathbf{R}$ is infinite. (2)

7) a) Find the roots of the polynomial equation

$$x^4-9x^3+17x^2+33x-90=0,$$

given that two of its roots are equal to 3. (3)

b) Use the principle of mathematical induction to show that for any positive integer $n, (3^n-2^n)>0$. (3)

c) A coach buys 3 cricket bats and 6 balls for ` 3,900. Later, he buys 1 bat and 3 balls for another team for ` 1,450. Write two linear equations to represent the purchases. Why can the linear system so obtained be solved by Cramer's rule? Also, use the rule to solve the system, and interpret the solution in the given context. (4)

8) a) Use Cardano's method to obtain the roots of $x^3 - 3x + 2 = 0$. (5)

b) Show that $\frac{x^n - 1}{x - 1} \geq n x^{\frac{1}{2}(n-1)}, x \geq 0, x \neq 1, n \in \mathbf{N}$. (5)

9) a) If A, B are any two sets, then state the conditions under which $A \times B = B \times A$. Justify your conditions. (2)

b) Obtain the resolvent cubic of $x^4 + 5x^3 - 10x + 2 = 0$, according to Ferrari's method. (4)

c) Give a real life situation problem, which is mathematically translated into
$2x + y + 2z = 18, x + 3y + 3z = 24, 3y = 6$
Also, explain how this linear system models your problem. (4)