MTE-13

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

Discrete Mathematics

(Valid from 1st July, 2011 to 31st March 2012.)

It is compulsory to submit the assignment before filling in the exam form.

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi
(For July, 2011 cycle)

Dear Student,
Please read the section on assignments in the Programme Guide for elective Courses that we sent you after your enrolment. As you may know already from the programme guide, the continuous evaluation component has 30% weightage. This assignment is for the continuous evaluation component of the course.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO : .....

NAME : .....

ADDRESS : .....

.....

.....

COURSE CODE : ....

COURSE TITLE : .....

STUDY CENTRE : ..... **DATE** .....

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is valid only up to March, 2012. If you fail in this assignment or fail to submit it by March, 2012, then you need to get the assignment for the January 2012 cycle and submit it as per the instructions given in the Programme Guide.

We **strongly** suggest that you retain a copy of your answer sheets.

Wish you good luck.

Assignment

Course Code: MTE-13
Assignment Code: MTE-13/TMA/2011-12
Maximum Marks: 100

1) Which of the following statements are true and which are false? Give reasons for your answer.

   i) The contrapositive of 'If Maya works outside the house, then she will be independent.' is 'If Maya works outside the house, she will not be independent.'.

   ii) The number of onto functions from $\{1,2,3,4,5,6\}$ to $\{a,b,c,d\}$ is $4!S_6^4$.

   iii) If $k > 0$, the exponential generating function of the sequence

   $$\frac{(n+k-1)!}{(k-1)!}, \qquad n \geq 0$$

   is $(1-x)^{-k}$.

   iv) Any connected graph with 6 vertices and 8 edges is planar.

   v) Every bipartite graph with odd number of vertices is non-hamiltonian. (10)

2) a) Which of the following sentences are statements? Give reasons for your answer.

   i) Are the buses running today?

   ii) What a pleasant weather!

   iii) $x^2 + 1 = 0$ for a real value of x.

   iv) Every odd number is a prime. (4)

   b) Let $n \in N$ be fixed. For $0 \leq r \leq n$, let $\#_n(r)$ denote the number of subsets of $\{1,2,\ldots,n\}$ with r elements. Find a first order recurrence, with an initial condition for the sequence $\{\#_n(r)\}_{r=0}^{n}$. (3)

   c) Show that any tree with exactly two vertices of degree 1 is a path. (3)

3) a) Let $\mathscr{F}_n$ denote the nth Fibonacci number. Show by induction that every natural number is expressible as a sum

   $$\mathscr{F}_{n_1} + \mathscr{F}_{n_2} + \cdots + \mathscr{F}_{n_k}$$

   where $n_i - n_{i+1} > 1$ for each $i \geq 1$. (4)

   b) Find the number of ways of tying up 7 different books into 4 bundles if the order of the books in the bundles does not matter. (4)

   c) Give examples of trivially true and vacuously true statements. (2)

4) a) Write $(x \wedge z) \vee (y \vee z')$ in CNF and DNF. (7)

   b) A die is rolled twice and the sum of the numbers that appear is observed. What is the probability that the sum is either a perfect square or a perfect cube? (3)

5) a) Make a table of the values of the Boolean function

$$f(x_1, x_2, x_3) = x_2 \oplus (x_1 \vee x_3)$$

Write the function in DNF using the table. (5)

b) Find the general form of the solution to a linear homogeneous recurrence with constant coefficients for which the characteristic roots are 1 with multiplicity 1, -2 with multiplicity 2 and 2 with multiplicity 3. The relation also has a non-homogeneous part which is a linear combination of $n2^n$ and $(-2)^n$. (5)

6) a) Let A be an 8×8 Boolean matrix (i.e. every entry is 0 or 1). If the sum of the entries in A is 51, prove that there is a row i and a column j in A such that the entries in row i and in column j add up to more than 13. Further, show that there are at least 4 such pairs of rows and columns. (4)

b) If a planar graph has the degree sequence $\{2,2,3,3,4,4,4\}$, how many faces will it have? Draw a planar graph with this degree sequence and number the faces to check your answer. (4)

c) Give the order and the degree of the recurrence

$$a_{n+2}^3 = a_n^2 + 2a_n a_{n-1}^3 + 4n$$

Is the recurrence homogeneous? (2)

7) a) Solve the recurrence

$$6a_{n+3} - 13a_{n+2} + 9a_{n+1} - 2a_n = 2^{-n}, n \geq 0, \text{if } a_0 = 1, a_1 = 1, a_2 = 1$$ (5)

b) How many numbers from 0 to 999(0 and 999 inclusive) are not divisible by 7 or 11. (3)

c) If an apple costs Rs. 20, a papaya costs Rs. 15 and a banana costs Rs. 2, write the generating function for the number of ways of purchasing these fruits with Rs. n. (2)

8) a) Consider the weighted complete graph in Fig. 1. Start with the Hamiltonian Cycle $\{a,b,c,d,e\}$ and carry out the reduction step to get a cycle of lesser weight. (3)

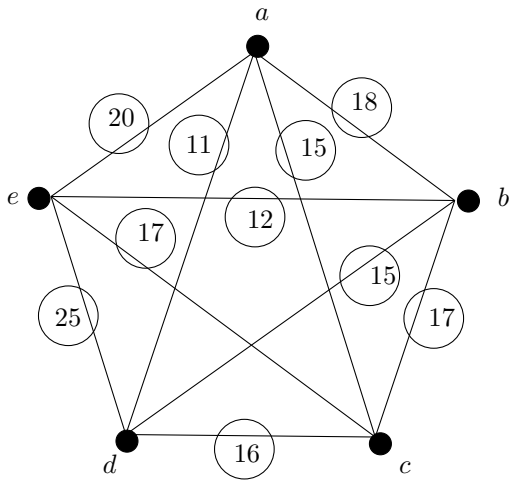

Figure 1

b) Find the generation function of the recurrence

$$a_n = 4a_{n-1} - 4a_{n-2} + 1$$

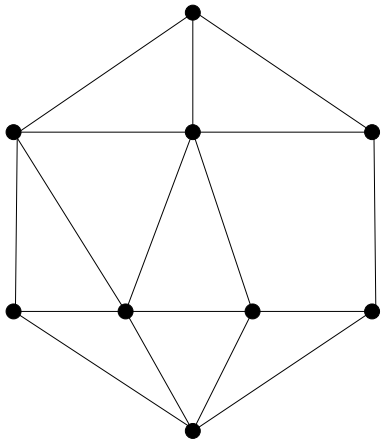

Figure 2

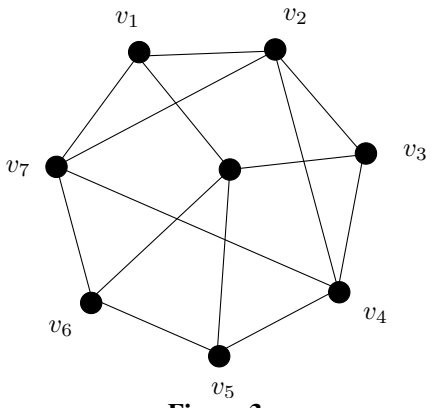

Figure 3

with initial conditions $a_0 = 2$, $a_1 = 5$. (4)

c) Express $4x^4 + 2x^3 + 2x^2 + x$ in terms of $[x]_4$, $[x]_3$, $[x]_2$ and $[x]$. (3)

9) a) Obtain the Ferrar graph of the partition $8+7+6+5+5+3+2+1$. Find the conjugate partition. Is the partition self conjugate? (3)

b) For the graph in Fig. 2 find a minimal colouring. (5)

c) Is Petersen graph bipartite? Give reasons for your answer. (2)

10) a) Find the number of equivalence relations that can defined on a set of 6 elements. (2)

b) Consider the graph given in Fig 3:

i) Write the degree sequence of the graph.

ii) Exhibit a cycle of length 9 in this graph.

iii) Does this have any complete graph as its subgraph? If yes, exhibit one.

iv) Find $\delta(G)$ and $\Delta(G)$ for this graph. (3)

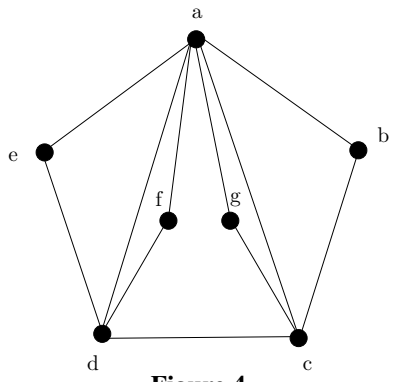

Figure 4

c) Verify that the graph in Fig. 4 is Eulerian. Use Fleury's algorithm to find an Eulerian circuit starting from a. You should explicitly mention the bridges that you used. (5)