MMT-009

ASSIGNMENT BOOKLET

(Valid from 1st January, 2025 to 31st December, 2025)

M.Sc. (Mathematics with Applications in Computer Science)
MATHEMATICAL MODELLING (MMT-009)

School of Sciences
Indira Gandhi National Open University
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New Delhi-110068
(2025)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO :……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO. …………………………..…

STUDY CENTRE: ………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is **valid from 1st Jan, 2025 to 31st Dec, 2025**. If you have failed in this assignment or fail to submit it by Dec, 2025, then you need to get the assignment for the year 2026, and submit it as per the instructions given in the Programme Guide.

7) **You cannot fill the examination form for this course** until you have submitted this assignment.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment

Course Code: MMT-009
Assignment Code: MMT-009/TMA/2025
Maximum Marks: 100

1. a) Companies located on the banks of a river, dumping their chemicals waste into river, causing high levels of pollution. Local authorities passed new legislation with very high fines if the pollution in the river exceeds certain specified concentration limits. State, giving reasons, the type of modeling you will use to find a policy for discharging the waste to ensure that the concentration level never exceeds the specified limits. Also state four essentials and two non-essentials for the problem. (6)

   b) Consider the following data (4)

x	2	3	4	5	6
y	8.3	16.5	30.2	65.2	125.6

   Use a best fit line to estimate the value of y when $x = 4.5$.

2. a) Five securities have the following expected returns (4)

   $A = 20\%, B = 15\%, C = 25\%, D = 22\%, E = 18\%$. Calculate the expected returns for a portfolio consisting of all five securities under the following conditions

   i) The portfolio weights are of equal percentage in each
   ii) The portfolio weights are 32% in A and remaining are equally divided among other four securities.

   b) Let $P = (w_1, w_2)$ be a portfolio of two securities. If variance of P is minimum then find the value of w_1 and w_2 in the following situations. (6)

   i) $\rho_{12} = -1$
   ii) $\sigma_1 = \sigma_2$
   iii) $\rho_{12} = -0.5, \sigma_1 = 1.5$ and $\sigma_2 = 2.5$.

3. a) Assume that the return distribution on the two securities X and Y be as given below: (6)

	Market 1	Market 2	Market 3
Probability	0.2	0.5	0.3
Security X	-20%	18%	40%
Security Y	-10%	20%	15%

   which security is more risky in the Markowitz sense. Also find the correlation coefficient of securities X and Y.

   b) In a species of animals a constant fraction of the population $\alpha = 6.2$ are born each breeding season and a constant fraction $\beta = 4.5$ die. Formulate a difference equation for the population and find out the number of individuals after fifteen seasons given that the initial number is 987. Find the closed form solution of the formulated difference equation. If the growth rate of the population is represented by r then interpret the solution obtained when i) $r > 0$ and ii) $r < 0$. (4)

4. A model for insect populations leads to the difference equations (10)

$$N_{k+1} = \frac{\lambda N_k}{1 + aN_k}$$

where λ and a are positive constants.

i) Write the equation in the form $N_{k+1} = N_k + R(N_k)N_k$ and hence identify the growth rate.

ii) Plot the graph of $R(N_k)$ as a function of N_k.

iii) Express the intrinsic growth rate r and the carrying capacity K, for this model, in terms of the parameters, a and λ.

iv) Find the steady-state solution of this model and analyse the solution.

5. Do the stability analysis of the following competing species system of equations with diffusion and advection (10)

$$\frac{\partial N_1}{\partial t} = a_1 N_1 - b_1 N_1 N_2 + D_1 \frac{\partial^2 N_1}{\partial x^2} - V_1 \frac{\partial N_1}{\partial x}$$
$$\frac{\partial N_2}{\partial t} = -d_1 N_2 + C_1 N_1 N_2 + D_2 \frac{\partial^2 N_2}{\partial x^2} - V_2 \frac{\partial N_2}{\partial x}, 0 \le x \le L$$

where V_1 and V_2 are advection velocities in x direction of the two populations with densities N_1 and N_2 respectively. a_1 is the growth rate, b_1 is the predation rate, d_1 is the death rate, C_1 is the conversion rate. D_1 and D_2 are diffusion coefficients. The initial and boundary conditions are:

$$N_i(x,0) = f_i(x) > 0, 0 \le x \le L, \ \ i = 1,2...$$
$$N_i = \overline{N}_i \text{ at } x = 0 \text{ and } x = L \ \forall \ t, i = 1,2...$$

where $\overline{N}_i$ are the equilibrium solutions of the given system of equations. Interpret the solution obtained and also write the limitations of the model.

6. The population dynamics of a species is governed by the discrete model (10)

$$x_{n+1} = x_n \exp\left[r\left(1 - \frac{x_n}{K}\right)\right],$$

where r and k are positive constants. Determine the steady states and discuss the stability of the model. Find the value of r at which first bifurcation occurs. Describe qualitatively the behaviors of the population for $r = 2 + \varepsilon$, where $0 < \varepsilon << 1$. Since a species becomes extinct if $x_n \le 1$ for any $n > 1$, show using iterations, that irrespective of the size of $r > 1$ the species could become extinct if the carrying capacity $k < r \exp\ [1 + e^{r-1} - 2r]$.

7. Do the stability analysis of the following model formulated to study the effect of toxicant on one competing species where the environment toxicant concentration is being taken to change w.r.t. time. (10)

$$\frac{dN_1}{dt} = r_1 N_1 - \alpha_1 N_1 N_2 - d_1 C_0 N_1$$

$$\frac{dN_2}{dt} = r_2N_2 - \alpha_2 N_1 N_2$$
$$\frac{dC_0}{dt} = k_1P - g_1C_0 - m_1C_0$$
$$\frac{dP}{dt} = Q - hP - kPN_1 + gC_0N_2.$$

along with the initial conditions.

$$N_1(0) = N_{10}, N_2(0) = N_{20}, C_0(0) = 0, P(0) = P_0 > 0$$

Here,

$N_1(t) =$ Density of prey population
$N_2(t) =$ Density of predator population
$C_0(t) =$ Concentration of the toxicant in the individual of the prey population
$P =$ Constant environmental toxicant concentration.

α_1, α_2 are the predation rates, r_1, r_2 are the growth rates or birth rates, d_1 is the death rate due to C_0, m_1 is the depuration rate, Q, h, k, g are positive rate constants.

8. The owner of a readymade garments store sells two types of shirts: Zee-shirts and Button-down shirts. He makes a profit of Rs. 5 and Rs. 10 per shirt on Zee-shirts and Button-down shirt, respectively. He has two tailors, A and B at his disposal to stitch the shirts. Tailors A and B can devote at the most 7 hours and 15 hours per day, respectively. Both these shirts are to be stitched by both the tailors. Tailors A and B spend 2 hours and 5 hours, respectively in stitching one Zee-shirts, and 4 hours and 3 hours, respectively in stitching a Button-down shirt. How many shirts of both types should be stitched in order to maximize daily profit? (10)

   a) Formulate and solve this problem as an LP problem.

   b) If the optimal solution is not integer-valued, use Gomory technique to derive the optimal integer solution.

9. a) A company has three factories that supply to three markets. The transportation costs from each factory to each market are given in the table. Capacities of the factories and market requirements are shown. Find the minimum transportation cost. (6)

	M_1	M_2	M_3	a_i
F_1	2	1	3	20
F_2	1	2	3	30
F_3	2	1	2	10
b_j	10	10	20	40/60

   b) For a multi-channel queuing system with $\lambda = 12/\text{hours}$, $\mu = 5/\text{hours}, c = 3, p_0 = 0.056$, calculate (4)

      i) The average time a customer is in the system

      ii) The average number of customers in the system

iii) Whether any time would be saved for customers if the three-channel system with the service rate of 5 per hour is replaced by a single-channel system with an average service rate of 15 per hour?

10. a) Ships arrive at a port at the rate of one in every 4 hours with exponential distribution of inter-arrival times. The time a ship occupies a berth for unloading has exponential distribution with an average of 10 hours. If the average delay of ships waiting for berths is to be kept below 14 hours, how many berths should be provided at the port? (4)

b) A library wants to improve its service facilities in terms of the waiting time of its borrowers. The library has two counters at present and borrowers arrive according to Poisson distribution with arrival rate 1 every 6 minutes and service time follows exponential distribution with a mean of 10 minutes. The library has relaxed its membership rules and a substantial increase in the number of borrowers is expected. Find the number of additional counters to be provided if the arrival rate is expected to be twice the present value and the average waiting time of the borrower must be limited to half the present value. (6)