MMT-004

ASSIGNMENT BOOKLET

(Valid from 1st January, 2025 to 31st December, 2025)

M.Sc. (Mathematics with Applications in Computer Science)

REAL ANAYLSIS (MMT-004)

School of Sciences
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Maidan Garhi, New Delhi-110068
(2025)

Dear Student,

Please read the section on assignments in the Programme Guide that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO :……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO. ………………………….…

STUDY CENTRE: …………………………..….. DATE: ……………………….……………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is **valid from 1st Jan, 2025 to 31st Dec, 2025**. If you have failed in this assignment or fail to submit it by Dec, 2025, then you need to get the assignment for the year 2026, and submit it as per the instructions given in the Programme Guide.

7) **You cannot fill the examination form for this course** until you have submitted this assignment.

We strongly suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment (MMT – 004)

Course Code: MMT-004
Assignment Code: MMT-004/TMA/2025
Maximum Marks: 100

1. State whether the following statements are true or false. Justify your answers. 5×2=10

   a) The outer measure m^* of the set $A = \{x \in \mathbb{R} : x^2 = 1\} \cup [-3, 2]$ is 0.

   b) A finite subset of a metric space is totally bounded.

   c) A connected subspace in a metric space which in not properly contained in any other connected subspace is always open.

   d) The surface given by the equation $x + y + z - \sin(xyz) = 0$ can also be described by an equation of the form $z = f(x, y)$ in a neighbourhood of the point $(0,0)$.

   e) A real valued function f on $[a,b]$ is continuous if it is integrable on $[a,b]$.

2. a) Find the interior, closure, the set of limit points and the boundary of the set (3)

   $$A = \{(x, y) \in \mathbb{R}^2 : y = 1\}$$

   in $\mathbb{R}^2$ with the standard metric.

   b) Consider $f : \mathbb{R}^3 \to \mathbb{R}^3$ given by (4)

   $$f(x, y, z) = (2x + 3y + z, xy, yz, xz)$$

   Find $f(2, 0, -1)$.

   c) Does Cantor's intersection theorem hold for the metric space $X = (0,1]$ with the standard metric? Justify your answer. (3)

3. a) Obtain the second Taylor's series expansion for the function given by (3)

   $$f(x, y) = xy^2 + 5xe^y \text{ at } \left(1, \frac{\pi}{2}\right).$$

   b) Find the Lebesgue integral of the function f given by (3)

   $$f = \chi_{[0,1]} + 2\chi_{[3,5]} + \chi_{[6,9]}.$$

   c) Find and classify the extreme values of $(x, y) = xy$ (4)

   Subject to the constraint

   $$\frac{x^2}{8} + \frac{y^2}{2} - 1 = 0.$$

4. a) Let $f:\mathbb{R}^3\setminus\{(0,0,0)\}\to\mathbb{R}^3\setminus\{(0,0,0)\}$ be given by (4)

$$f(x,y,z)=\left(\frac{x}{x^2+y^2+3^2},\frac{y}{x^2+y^2+z^2},\frac{z}{x^2+y^2+z^2}\right)$$

Show that f is locally invertible at all points in $\mathbb{R}^3\setminus\{(0,0,0)\}$.

b) For the equation $x^2+y^2+z^3=0$, at which points on its solution set, can we assured that there is a neighbourhood of the point in which the surface given by the equation can be described by an equation of the form $z=f(x,y)$. (3)

c) Find the Fourier series of $f(t)=t^2$ on $[-\pi,\pi]$. (3)

5. a) Prove that if an open set $\bigcup$ can be written as the union of pariwise disjoint family V of open connected subsets, then these subsets must be the components of U. Use this theorem to find the components of the set $D\bigcup E$ where (6)

$$D=\{(x,y)\in\mathbb{R}^2:\|(x+1,y)\|=1\}\text{ and}$$
$$E=\{(x,y)\in\mathbb{R}^3:\|x-1,y\|=1\}.$$

b) Which of the following subsets of $\mathbb{R}$ are compact w.r.t. the metric given against them. Justify your answer. (4)

i) $A=(0,1)$ in $\mathbb{R}$ of $-\mathbb{R}$ with standard metric.
ii) $A=[3,4]-\mathbb{R}$ with discrete metric.
iii) $\{(x,y)\in\mathbb{R}^2 \quad y>0\}-\mathbb{R}^2$ with standard metric.

6. a) If E is a subset of $\mathbb{R}$ with standard metric, then show that $(\overline{E})^C=(E^C)^0$. (4)

b) Show that a set A in a metric space is closed if and only if every convergent sequence in A converges to a point of A. (4)

c) Find the interior and closure of the set $\mathbb{Q}$ of rationals in $\mathbb{R}$ with standard metric. (2)

7. a) Let F be the function from $\mathbb{R}^2$ to $\mathbb{R}^2$ defined by (5)

$$F(x,y)=(x^2+y^2,xy).$$

Show that F is differentiable at $(1,2)$. Find the differential matrix of F.

b) Show that the function f defined by (3)

$$f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}, & if\ (x,y)\neq(0,0)\\ 0 & ,if\ (x,y)=(0,0)\end{cases}$$

is not differentiable at $(0,0)$. Does the partial derivatives of f exists at $(0,0)$? or any at any other point in $\mathbb{R}^2$? Justify your answer.

c) Is the continuous image of a Cauchy sequence a Cauchy sequence? Justify. (2)

8. a) Find the directional derivative of the function $f:\mathbb{R}^4 \to \mathbb{R}^4$ defined by (4)

$$f(x,y,z,w) = (x^2 y, xyz, x^2 + y^2, zw^2)$$

at the point $(1,2,-1,-2)$ in the direction $v = (1,0,-2,2)$.

b) Suppose that $f:\mathbb{R} \to \mathbb{R}^2$ is given by $f(t) = (t,t^2)$ and $g:\mathbb{R}^2 \to \mathbb{R}^3$ is given by $g(x,y) = (x^2, xy, y^2 - x^2)$. Compute the derivative of $g \circ f$. (4)

c) Find $B\left[2,\frac{1}{2}\right]$ in $\mathbb{R}$ where d is the metric given by $d(x,y) = \min\{1, |x-y|\}$. (2)

9. a) Give an example of a family f_1 of subsets of a set X which has finite intersection property. Justify your choice of example. (5)

b) Verify the hypothesis and conclusions of the Fatou's lemma for the sequence $\{f_n\}$ given by (5)

$$f_n(x) = 2n \quad \text{for } x \in \left(\frac{1}{2n}, \frac{1}{n}\right)$$
$$= 0 \quad \text{for } x \in \left(0, \frac{1}{2n}\right) \cup \left(\frac{1}{n}, 1\right)$$

10. a) Let (X,d) be a metric space and A be a subset of X. Show that $bdy(A) = Q$ if and only if A is both open and closed. (3)

b) Give an example of an algebra which is not a $\sigma-$ algebra. Justify your choice of examples. (3)

c) If E is a measurable set and f is a simple function such that $a \le f(x) \le b \ \forall x \in E$, show that (4)

$$am(E) \le \int_E f\, dm \le m(E)b.$$