MMTE-002

ASSIGNMENT BOOKLET

(Valid from 1st January, 2024 to 31st December, 2024)

M.Sc. (Mathematics with Applications in Computer Science)

DESIGN AND ANALYSIS OF ALGORITHMS

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi
(2024)

Dear Student,
Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 percent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO. : .....

NAME : .....

ADDRESS : .....

.....

.....

COURSE CODE : ....

COURSE TITLE : .....

STUDY CENTRE : ..... **DATE** .....

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Programme Centre as per the schedule made by the Programme Centre. Answer sheets received after the due date shall not be accepted.

   We **strongly** suggest that you retain a copy of your answer sheets.

7) This assignment is valid only up to December, 2024. If you fail in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So, solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MMTE-002
Assignment Code: MMTE-002/TMA/2024
Maximum Marks: 100

1) a) The maximum subsequence sum problem is defined as follows: If a_1, a_2, …, a_n are in **Z**, find the maximum value $\sum_{k=i}^{j} a_i$, for all i, j, $1 \leq i \leq j \leq n$. We assume that the answer is 0 if all the a_i are negative or if the sum is empty. The following algorithm finds a solution to the problem. Here, we assume that a_is are stored in the array A.

MAXIMUM-SUBSEQUENCE(A, $MaxSum$)

```
1  Sum ← 0, MaxSum ← 0
2  for i ← 1 to n
3       do
4            Sum = Sum+A[i]
5            if Sum > MaxSum
6              then MaxSum ← Sum
7              else  if Sum < 0
8                        then Sum = 0
```

State precisely a loop invariant for the **for** loop in line 2–8. Prove that your loop invariant holds and hence conclude that the algorithm works. (5)

b) Analyse the algorithm and find an upper bound for the run time of the above algorithm. (5)

2) a) With the help of an example, explain the following:
i) Algorithm.
ii) Input and output for an algorithm.
iii) Running time of an algorithm. (5)

b) Using Fig. 7.1 in page 147 of the book as the model, illustrate the operation of PARTITION on the array $A = \{7, 6, 9, 8, 17, 12, 5, 11, 13, 14, 12\}$ (5)

3) a) For the set of keys $\{3, 7, 9, 4, 6, 8, 12\}$ draw binary search trees of height 2, 3, 4, 5 and 6. (5)

b) Using Fig. 6.3 in page 134 of the book as a model, illustrate the operation of BUILD-MAX-HEAP on the array $A = \{6, 7, 4, 9, 13, 11, 8, 12, 5\}$. (5)

4) a) Show the results of inserting the keys

$$Q, S, F, K, H, L, F, T, V, P, M, R, N, W, A$$

in order into an empty B-tree with minimum degree 2. Only draw the configurations of the tree just before some node must split, and also draw the final configuration. (5)

b) Suppose the CONNECTED-COMPONENTS is run on the undirected graph $G = (V, E)$, where $V = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and the edges in $E(V) = \{e_1 = (1, 3), e_2 = (2, 5), e_3 = (3, 6), e_4 = (5, 8), e_5 = (5, 9), e_6 = (6, 9), e_7 = (4, 7), e_8 = (3, 8)\}$ are processed in the order $\{e_1, e_2, e_3, e_4, e_6, e_5, e_7, e_8\}$. List the vertices in each connected component after each iteration of lines 3–5 in the CONNECTED-COMPONENTS. (5)

5) a) Show how mergesort sorts the array $A = \{7, 9, 4, 12, 8, 6, 10, 5\}$ (3)

b) For the following set of points, describe how the CLOSEST-PAIR algorithm finds a closest pair of points: (3)

$$(3, 2), (2, 1), (2, 3), (1, 2), (3, 1), (2, 2), (1, 3), (3, -1), (5, -2)$$

c) Find an optimal parenthesisation of a matrix chain product whose sequence of dimensions is $(3, 5, 7, 3, 4)$. (4)

6) a) In the **Coin changing problem**, we have to give change for n rupees using the least number of coins of a given set of denominations. It is clear that we cannot give change for any n for all set of denominations. For example, trivially, we cannot give change for ₹. 3, if no 1, 2 or 3 rupees coins do not exist or not included in the allowed denominations. If the set of denominations include 1 ₹., then we can always give change, so that there is a way of changing n ₹. for any n. We can use greedy approach to find the optimal solutions for many set of denominations. Show that, however, there are set of denominations for which we cannot find the optimal solution by greedy approach. You should include 1 ₹. in your denominations so that a solution always exists for the problem. (5)

b) Determine an LCS of $(1, 0, 1, 1, 0, 0, 1, 0, 1)$ and $(0, 0, 1, 0, 1, 1, 0, 1, 0)$. (5)

7) a) Show the d and π values that result from running breadth-first search on the graph given in Fig. 1 using vertex 4 as the source. (7)

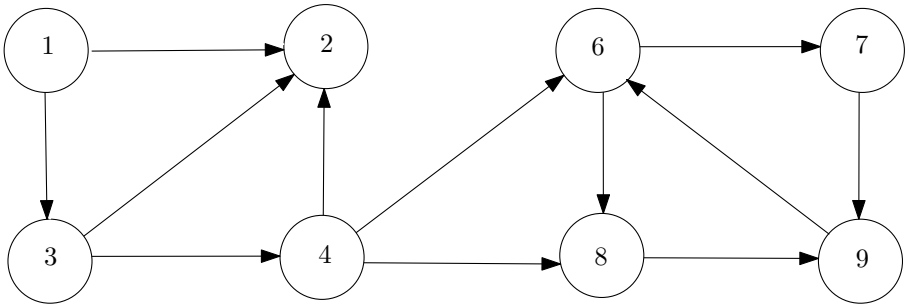

Figure 1: Figure for exercise 7(a).

b) Use Kruskal's algorithm to find a minimal spanning tree in the graph given in Fig. 2. (7)

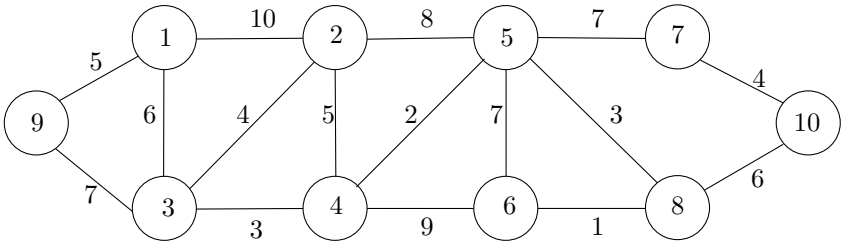

Figure 2: Figure for exercise 7(b)

c) Use Dijkstra's algorithm to find the shortest paths in the graph given in Fig. 3 with a as the source vertex. (6)

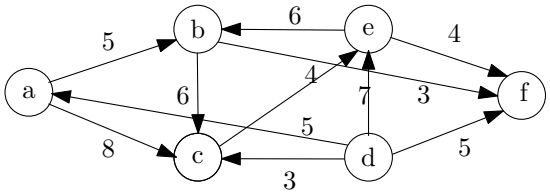

Figure 3: Figure for exercise 6(c).

8) a) Show the comparisons the naive string matcher makes for the pattern $P = 0100$ with 011000100101001000. (4)

b) When working modulo $q = 17$, how many spurious hits does the Rabin-Karp matcher encounter in the text $T = 29103292566473$ when looking for the pattern 22? (3)

c) Compute the values (d, x, y) that the call EXTENDED-EUCLID(10117,11591) returns. (3)

9) a) Find all the solutions of the equation $6x \equiv 4 \pmod{114}$. (3)

b) Let $\{(-1, -5), (0, -4), (1, -1)\}$ and $\{(-1, 14), (0, 7), (1, 4)\}$ be the point-value representations of two polynomials $f(x)$ and $g(x)$. Find the point-value representation of $h(x) = f(x) + g(x)$. From the point value representation of $h(x)$ find the coefficient representation of $h(x)$. (4)

c) Compute the DFT of the vector $(-1, 3, 1, -1)$. (3)