MMT-008

ASSIGNMENT BOOKLET

(Valid from 1st January, 2024 to 31st December, 2024)

M.Sc. (Mathematics with Applications in Computer Science)

PROBABILITY AND STATISTICS (MMT-008)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068

(2024)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective courses that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO :……………………………………………
NAME :……………………………………………
ADDRESS :……………………………………………
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COURSE CODE: ……………………………
COURSE TITLE : …………………………….
ASSIGNMENT NO. ……………………………
STUDY CENTRE: …………………………….. DATE: ……………………………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2024. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the session 2025 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment (MMT – 008)

Course Code: MMT-008

Assignment Code: MMT-008/TMA/2024

Maximum Marks: 100

1. a) Consider the Markov chain having the following transition probability matrix. (5)

$$p = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{5} & \frac{4}{5} & 0 & 0 \end{bmatrix} \end{array}$$

i) Draw the diagram of a Markov chain.

ii) Classify the states of a Markov chain, i.e., persistent, transient, non-null and a periodic state. Also check the irreducibility of Markov chain.

iii) Find the closed sets.

iv) Find the probability of absorption to the closed classes. Also find the mean time up to absorption from transient state 3 to 4.

b) Determine the parameters of the bivariate normal distribution: (5)

$$f(x,y) = k \exp\left[-\frac{8}{27}\left\{(x-7)^2 - 2(x-7)(y+5) + 4(y+5)^2\right\}\right]$$

Also find the value of k.

2. a) Suppose that the probability of a dry day (State 0) following a rainy day (State 1) is $\frac{1}{3}$ and the probability of a rainy day following a dry day is $\frac{1}{2}$. Write the transition probability matrix of the above Markov chain. (5)

Given that 1st May is a dry day, then calculate

i) the probability that 3rd May is also a dry day.
ii) the stationary probabilities.

b) Let $\underset{\sim}{X} \sim N_4(\underset{\sim}{\mu}, \Sigma)$ with

$$\underset{\sim}{\mu} = \begin{pmatrix} 2 \\ 1 \\ 3 \\ -4 \end{pmatrix} \text{ and } \Sigma \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & -2 & -1 \\ 1 & -2 & 9 & -1 \\ 1 & -1 & -1 & 16 \end{bmatrix}$$

Support $\underset{\sim}{Y}$ and $\underset{\sim}{Z}$ are two partitioned subvectors of $\underset{\sim}{X}$ such that $\underset{\sim}{Y}' = (x_1\ x_3)$ and $\underset{\sim}{Z}' = (x_2\ x_4)$

i) Obtain the marginal distribution of $\underset{\sim}{Y}'$.

ii) Check the independence of $\underset{\sim}{Y}'$ and $\underset{\sim}{Z}'$.

iii) Obtain the conditional distribution of $\underset{\sim}{Y}' | \underset{\sim}{Z}'$; where $\underset{\sim}{Y}' = (x_1\ x_2)$, $\underset{\sim}{Z}' = (x_3\ x_4)$.

iv) Find $E(\underset{\sim}{Y}' | \underset{\sim}{Z}')]$; where $\underset{\sim}{Y}'$ and $\underset{\sim}{Z}'$ are same as in (iii). (5)

3. a) Suppose that customers arrive at a service counter in accordance with a Poisson process with the mean rate 2 per minute. Then obtain the probability that the interval between two successive arrivals is (5)

i) more than 1 minute.

ii) 4 minutes or less.

iii) between 1 and 2 minutes.

b) Write two advantages and two disadvantages of conjoint analysis. (5)

4. a) Find the differential equation of pure birth process with $\lambda_K = K\lambda$ and the process start with one individual at time $t = 0$. Hence, find $p_n(t) = P(N(t) = n)$ [N(t) is the number present at time t] with $E(N(t))$ and $Var(N(t))$. Also identify the distribution. (5)

b) Let $\{X_n; n \geq 1\}$ be an i.i.d. sequence of interoccurrence times with common probability mass function given by

$$P(X_n = 0) = \frac{2}{3}, P(X_n = 1) = P(X_n = 2) = \frac{1}{6}.$$

Let $N_t; t \geq 0$ be the corresponding renewal process. Find the Laplace transform $\tilde{M}_t$ of the renewal function, M_t. (5)

5. The body dimensions of a certain species have been recorded. The information of body length L and body weight W are given below:

Body length L (in mm)	Body weight W (in mg)
45	2.9
48	2.4
45	2.8
48	2.9
44	2.4
45	2.3
45	3.1
42	1.7
50	2.4
52	3.7

At 5% level of significance, test the hypothesis that all variances are equal and all covariances are equal in variance-covariance matrix for the given data. (10)

[You may like to use the values, $\chi^2_{9,0.05} = 3.84, \chi^2_{10,0.05} = 4.10, \chi^2_{11,0.05} = 5.09$]

6. The Tooth Care Hospital provides free dental service to the patients on every Saturday morning. There are 3 dentists on duty, who are equally qualified and experienced. It takes on an average 20 minutes for a patient to get treatment and the actual time taken is known to vary approximately exponentially around this average. The patients arrive according to the Poisson distribution with an average of 6 per hour. The officer of the hospital wants to investigate the following: (10)

   i) The expected number of patients in the queue.

   ii) The average time that a patient spends at the clinic.

   iii) The average percentage idle time for each of the dentists.

7. a) For the two-state Markov chain, whose transition probability matrix is (5)

$$P = \begin{pmatrix} 1-p & p \\ p & 1-p \end{pmatrix}; 0 \le p \le 1.$$

Find all stationary distributions.

b) Let p_K, where $K = 0,1,2$ be the probability that an individual generates K offsprings. Then find the p.g.f. of $\{p_K\}$. Also, calculate the probability of extinction when (5)

i) $p_0 = \frac{1}{4}$, $p_1 = \frac{1}{4}$ and $p_2 = \frac{1}{2}$.

ii) $p_0 = \frac{2}{3}$, $p_1 = \frac{1}{6}$ and $p_2 = \frac{1}{6}$.

8. a) Let $p = 3$ and $m = 1$ and suppose the random variables X_1, X_2 and X_3 have the positive definite covariance matrix: (6)

$$\sum = \begin{bmatrix} 1 & 0.4 & 0.3 \\ 0.4 & 1 & 0.2 \\ 0.3 & 0.2 & 1 \end{bmatrix}$$

Write its factor model.

b) For X distributed as $N_3(\mu, \Sigma)$, find the distribution of (4)

$$\begin{bmatrix} X_1 & -X_2 & X_3 \\ -X_1 & X_2 & X_3 \end{bmatrix}.$$

9. a) The joint density function of random variables X, Y and Z is given as (5)

$$f(x,y,z) = K.x.e^{-(y+z)}; \text{ where } 0 < x < 2, y \ge 0 \text{ and } z \ge 0.$$

Find

i) the constant K.
ii) the marginal distributions of X, Y and Z.
iii) $E(X), E(Y)$ and $E(Z)$.
iv) the conditional expectation of Y given X and Z.
v) the correlation coefficient between X and Y.

b) For the model $M | M | 1 | N | FIFO$, calculate the steady state solution for P_0. (5)

$E(n)$ – Average number of customers in the system

E(V) – Average waiting time in the system

10. State which of the following statements are true and which are false. Give a short proof or a counter example in support of your answer. (10)

a) For three independent events E_1, E_2 and E_3,

$$P(E_1 \cup E_2 \cup E_3) + P(\overline{E}_1)P(\overline{E}_2)P(\overline{E}_3) = 0.$$

b) The range of multiple and partial correlation coefficient is $]-1,1[$.

c) If $\{X(t); t \geq 0\}$ is a poisson process, then $N(t) = [X(t+S_0) - X(t)]$ where $S_0 > 0$ is a fixed constant, is also a poisson process.

d) In Hotelling T^2, the value of S is given by

$$S = \frac{1}{n-1}\sum_{j=1}^{n}(X_j - \mu)(X_j - \mu)'.$$

e) Let $\underset{\sim}{X}_{p\times 1} \sim N_p(\mu, \Sigma)$ and $X_{p\times n}$ be the state matrix, then parameters involved in the above distribution are p for μ and $\frac{1}{2}p(p+1)$ for Σ.