MMT-007

ASSIGNMENT BOOKLET

(Valid from 1st January, 2024 to 31st December, 2024)

M.Sc. (Mathematics with Applications in Computer Science)
Differential Equations and Numerical Solutions (MMT-007)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi,
New Delhi-110068
(2024)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective courses that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO :……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO. ………………………….…

STUDY CENTRE: ………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved..

6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto December, 2024. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by December, 2024, then you need to get the assignment for the year 2025 and submit it as per the instructions given in the programme guide.

8) You can not fill the Exam Form for this course till you have submitted this assignment. So solve it and submit it to your study centre at the earliest.

We wish you good luck.

Assignment

Course Code: MMT-007
Assignment Code: MMT-007/TMA/2024
Maximum Marks: 100

1. a) Show that $f(x, y) = xy$

   i) satisfies a Lipschitz condition on any rectangle $a \le x \le b$ and $c \le y \le d$;

   ii) satisfies a Lipschitz condition on any strip $a \le x \le b$ and $-\infty < y < \infty$;

   iii) does not satisfy a Lipschitz condition on the entire plane. (4)

   b) Use Frobenious method to find the series solution about $x = 0$ of the equation

   $$x(1-x)\frac{d^2y}{dx^2} - (1+3x)\frac{dy}{dx} - y = 0. \qquad (6)$$

2. a) For the following differential equation locate and classify its singular points on the x-axis

   i) $x^3(x-1)y'' - 2(x-1)y' + 3xy = 0$

   ii) $(3x+1)xy'' - (x+1)y' + 2y = 0$ (2)

   b) Show that $L_{n+1}(x) = (2n+1-x)L_n(x) - n^2 L_{n-1}(x)$. (2)

   c) Show that $\displaystyle\int_{-1}^{1} x^2 P_{n-1}(x)\; P_{n+1}(x)\; dx = \frac{2n(n+1)}{(2n-1)\;(2n+1)\;(2n+3)}$. (3)

   d) Construct Green's function for the differential equation

   $$xy'' + y' = 0, \quad 0 < x < \ell$$

   under the conditions that $y(0)$ is bounded and $y(\ell) = 0$. (3)

3. a) Show that between every successive pair of zeros of $J_0(x)$ there exists a zero of $J_1(x)$. (2)

   b) Using the transformation $y = x^{1/2}u,\ 2x^{3/2} = 3z$ find the solution of the equation $y'' + x\ y = 0$ in terms of Bessel's functions. (3)

   c) Show that $\displaystyle\int_0^{\infty} e^{-ax}\; J_0(bx)\; dx = \frac{1}{\sqrt{a^2+b^2}}$, $a > 0\ b > 0$. (5)

4. a) Find the Laplace transform of $\dfrac{\cos\sqrt{t}}{\sqrt{t}}$. (4)

   b) If k_m and k_n are distinct roots of Bessel function $J_p(kb) = 0$ with $p \ge 0,\ b > 0$ then show that

   $$\int_0^b x\; J_p(k_m x)\; J_p(k_n x)\, dx = \begin{cases} 0 & \text{if} \quad m \ne n \\ \dfrac{b^2}{2}\left[J_{p+1}(k_n b)\right] & \text{if} \quad m = n \end{cases}. \qquad (6)$$

5. a) Solve the following IBVP using the Laplace transform technique:

$$u_t = u_{xx},\ \ 0 < x < 1,\ \ t > 0.$$
$$u(0,\ t) = 1,\ u(1,\ t) = 1,\ \ t > 0$$
$$u(x,\ 0) = 1 + \sin \pi x,\ \ 0 < x < 1. \qquad (5)$$

b) If the Fourier cosine transform of $f(x)$ is $\alpha^n e^{-a\alpha}$, then show that

$$f(x) = \frac{2}{\pi}\ \frac{n!\cos(n+1)\theta}{(a^2 + x^2)^{\frac{n+1}{2}}}. \qquad (5)$$

6. a) Find the displacement $u(x,\ t)$ of an infinite string using the method of Fourier transform given that the string is initially at rest and that the initial displacement is $f(x),\ -\infty < x < \infty$. (6)

b) Using Fourier integral representation show that

$$\int_0^\infty \frac{\cos(\alpha x) + \alpha \sin(\alpha x)}{1+\alpha^2} d\alpha = \begin{cases} 0 & \text{if} \quad x < 0 \\ \pi/2 & \text{if} \quad x = 0. \\ \pi e^{-x} & \text{if} \quad x > 0 \end{cases} \qquad (4)$$

7. a) Using Runge-Kutta second order method with

(i) $h = 0.1$, (ii) $h = 0.2$, solve the initial value problem
$y' = y^2 \sin x,\ \ y(0) = 1$.

Upto $x = 0.4$. If the exact solution is $y = \sec x$, obtain the error. (5)

b) Solve the heat conduction equation $u_t = u_{xx}$ in the region $R(0 \le x \le 1,\ \ t > 0)$ with the initial and boundary conditions $u(x,\ 0) = 0,\ u(0,\ t) = 0,\ u(1,\ t) = t$ using Crank-Nicolson method with $h = 0.25$ and $\lambda = 1$ upto two time steps. (5)

8. a) Using second order finite Difference method, solve the boundary value problem
$y'' + 5y' + 4y = 1,\ \ y(0) = 0,\ y(1) = 0,\ h = 1/4$. (3)

b) Solve the wave equation $u_{tt} = u_{xx}$ with the initial and boundary conditions
$u(x,\ 0) = 0,\ u_t(x,\ 0) = 0,\ u(0,\ t) = 0,\ u(1,\ t) = 100 \sin \pi t$.
with $h = k = 0.25$, using the explicit method upto four time levels. (7)

9. a) Find an approximate value of $y(1.0)$ for the initial value problem

$$y' = x^3 - y^3,\ \ y(0) = 1$$

using the multiple method

$$y_{n+1} = y_n + \frac{h}{3}\ [7f_n - 2f_{n-1} + f_{n-2}]$$

with step length $h = 0.2$. Calculate the starting values using Runge-Kutta second order method with the same h. (5)

b) Using standard five point formula, solve the Laplace equation $\nabla^2 u = 0$ in R where R is the square $0 \le x \le 1,\ 0 \le y \le 1$ subject to the boundary conditions $u(x,\ y) = x^2 - y^2$ on $x = 0,\ y = 0,\ y = 1$ and $3u + 2\frac{\partial u}{\partial x} = x^2 + y^2$ on $x = 1$. Assume $h = k = 1/2$. (5)

10\. a) Find an approximate value of $y(1.0)$ for the initial value problem

$$y' = x - 2y, \quad y(0) = 1$$

using Milne-Simpson's method

$$y_{n+1} = y_{n-1} + \frac{h}{3}\left[f_{n+1} + 4f_n + f_{n-1}\right]$$

with the step length $h = 0.2$. Calculate the starting value using Runge-Kutta fourth order method with the same h. (6)

b) Using fourth order Taylor series method with $h = 0.2$, solve the initial value problem

$$y' = x + \cos y, \quad y(0) = 0$$

upto $x = 1$. (4)