MMTE-001

ASSIGNMENT BOOKLET

M.Sc. Mathematics with Applications in Computer Science (MSCMACS)

GRAPH THEORY

(1st January, 2023 to 31st December, 2023)

School of Sciences
Indira Gandhi National Open University
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(2023)

Assignment

(To be done after reading the course material)

Course Code: MMTE-001
Assignment Code: MMTE-001/TMA/2023
Maximum Marks: 100

1. State whether the following statements are **true** or **false**. Justify your answers with a short proof or a counterexample (20)

   i) Every tree has a perfect matching.

   ii) Every 2-connected bipartite graph is Hamiltonian.

   iii) For some nonnegative integers $d_1, d_2, \ldots, d_n$ the sequence $(\max\{d_1, n\}, d_2, d_3, \ldots, d_n)$ is graphic.

   iv) Grötzsch graph is Eulerian.

   v) K_4, as a plane graph, is self-dual.

   vi) The vertex-covering number of an odd cycle is 1 more than its independence number.

   vii) The complement of a disconnected graph is connected.

   viii) If $\kappa(G) < \kappa'(G)$, then $\delta(G) \geq 4$.

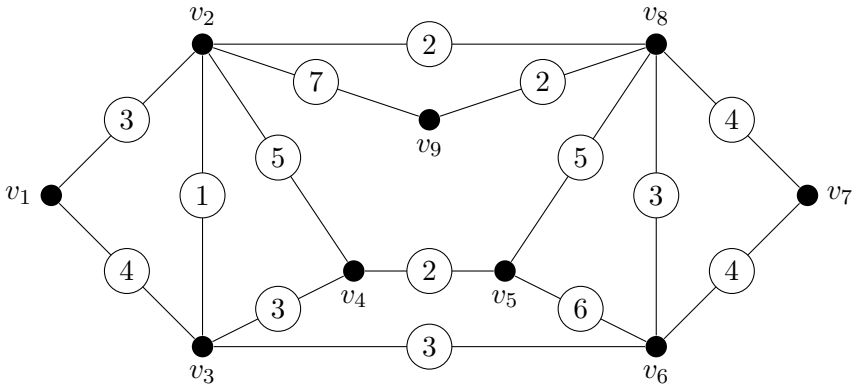
   ix) The line graph of the Petersen graph has 30 edges.

   x) There exists a complete binary tree on 15 vertices.

2. (a) Does there exist a 3-edge-colourable graph on 10 vertices and having 20 edges? Justify. (3)

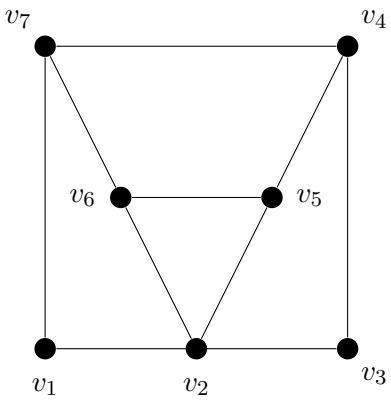
   (b) Prove or disprove that the height of an n-vertex complete k-ary tree is at least $\log_k(n+1) - 1$. (3)

   (c) Find a minimum-weigh spanning tree in the following graph. (4)



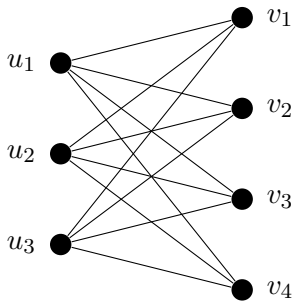
3. (a) An n-vertex forest with $n/2$ edges has exactly $n/2$ trees as its components. True or false? Justify. (3)

   (b) Draw the complement of the following graph.



Is the complement Hamiltonian? Justify your answer. (3)

(c) Verify the König Egarváry Theorem for the following graph. (4)



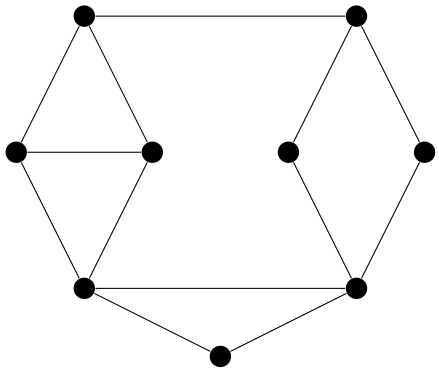
4. (a) Draw a diagram, as nice as possible, of the line graph of the Petersen graph. Write the number of vertices, the number of edges, the minimum and maximum degrees of it. (5)

(b) There exists a self-complementary graph on 2023 vertices. True of false? Justify your answer. (2)

(c) Every 3-colourable graph contains an odd cycle. True or false? Justify. (3)

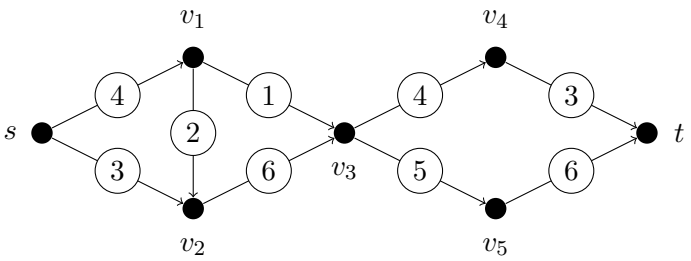
5. (a) If G is a k-connected graph having n vertices, what is the minimum size of G? Justify. (3)

(b) Draw the dual of the following plane graph.



Does the dual have any cut-vertex? Justify. (4)

(c) Define a flow on the following network, having value at least 5. (4)



6. (a) Check whether the sequence $(4, 4, 4, 3, 2, 2, 1, 1, 1)$ is graphic or not. If yes, draw a graph realising this degree sequence. (4)

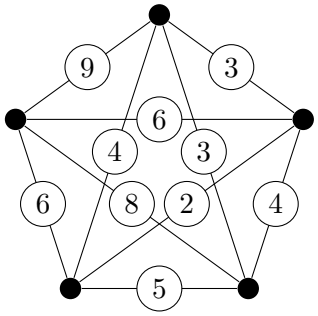
   (b) Let G be a graph having no isolated vertex and no induced subgraph with exactly two edges. Show that G is a complete graph. (6)

7. (a) Draw an $(8, 15)$-graph G with $\chi'(G) = 5$. (3)

   (b) Explain the difference between a maximal and a maximum matching, with the help of an example. (3)

   (c) Find the thickness of the line graph of K_4. (4)

8. (a) Starting with the cycle $(v_1, v_2, v_3, v_4, v_5, v_1)$ in the following weighted K_5 perform the reduction step twice to get a Hamiltonian cycle with smaller weight. (6)



   (b) Let G be a planar graph with at least 11 vertices. Show that $\overline{G}$ is nonplanar. (4)

9. (a) Let G and H be any graphs such that $L(G) \cong L(H)$. Is it necessary that $G \cong H$? Justify. (4)

   (b) For the graph given in Q. 3(b), find the number of (v_2, v_5)-walks of length 3. (6)