MMT-009

ASSIGNMENT BOOKLET

(Valid from 1st January, 2023 to 31st December, 2023)

M.Sc. (Mathematics with Applications in Computer Science)
Mathematical Modelling (MMT-009)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
2023

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective courses that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO :……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO. ………………………….…

STUDY CENTRE: …………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved..

6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto December, 2023. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by December, 2023, then you need to get the assignment for the year 2024 and submit it as per the instructions given in the programme guide.

We wish you good luck.

Assignment (MMT-009)

Course Code: MMT-009
Assignment Code: MMT-009/TMA/2023
Maximum Marks: 100

1. a) Let $P(t)$, measured in kg, be the total mass or biomass of the fish population in a point at time t. Write the continuous model for the population growth using logistic equation. The intrinsic growth rate r and the carrying capacity k are given the values 0.70 per year and 80.7×10^6 kg respectively. If the initial biomass is $P_0 = 0.25$ K, find the biomass after 2 years. Also find the time t, for which $P(t_1) = 0.75$ K. (5)

   b) A locality is served by two malls. Each mall has two counters to serve the customers. Both the malls are equally popular and are known to have equal shares of the market. This is evident from the fact that customer's arrive at each mall's serving counter at the rate of 12 customers per hour. The average time to serve a customer is 05 minutes. Customers' arrival is according to a Poisson distribution and the service time is exponential. To provide better service to the customers, the owners of the two malls decide to consolidate into a single larger mall. What is the effect of consolidation on the waiting time of customers? (5)

2. a) The transportation cost of 600 tons of a certain type of material from four factories B_1, B_2, B_3 and B_4 to three target stores T_1, T_2 and T_3 are given in the following table:

$$\begin{array}{c c} & \begin{array}{ccc} T_1 & T_2 & T_3 \end{array} \\ \begin{array}{c} B_1 \\ B_2 \\ B_3 \\ B_4 \end{array} & \begin{bmatrix} 8 & 6 & 5 \\ 6 & 6 & 6 \\ 10 & 8 & 4 \\ 8 & 6 & 4 \end{bmatrix} \end{array}$$

   The daily capacity of each of the factory is 150 per day and the daily requirement over each target store is 200. Find the allocation for each factory to each target store which minimize the total transport cost. (5)

   b) The return distribution on the two securities X and Y are given in the table below:

Possible Rates of Return		**Associated Probability**
X	Y	$P_{xj} = P_{yj}$
0.10	0.09	0.20
0.11	0.11	0.22
0.17	0.16	0.25
0.19	0.18	0.33

   Find σ_{XY} and ρ_{XY}. (5)

3. a) "Indifference curves of an investor cannot intersect." Is this statement true? Give reason for your answer. (3)

   b) Following is the data for number of years students studied a subject and score he/she received in that subject:

Number of Years	Test Score
3	57
4	78
4	72
2	58
5	89
3	63
4	73
5	84
3	75
2	48

Fit the least square line to this data. What is the score of the student who has studied for two years according to this line? (4)

c) Find the number of quantities required for estimating the expected return and standard deviation for 250 securities in Markowitz model. How many estimates are required for the securities while using single-index Sharpe model? (3)

4. a) Consider the discrete time population model given by $N_{t+1} = \dfrac{rN_t}{1+\left(\dfrac{N_t}{K}\right)^b}$ for a population, where r is the intrinsic growth rated, b is a positive parameter. Determine the non-negative steady-state and discuss the linear stability of the model for $9 < r < 1$. Also find the first bifurcation value of the parameters. (5)

b) Ships arrive at a port at the rate of one in every 4 hours with exponential distribution of inter arrival times. The time a ship occupies a berth for unloading has exponential distribution with an average of 10 hours. If the average delay of ships waiting for a berth is to be kept below 14 hours, how many berths should be provided at the port? (5)

5. a) A company has factories at F_1, F_2 and F_3 that supply products to warehouses at W_1, W_2 and W_3. The weekly capacities of the factories are 200, 160 and 90 units, respectively. The weekly warehouse requirements are 180, 120 and 150 units, respectively. The unit shipping costs (in ₹) are as follows:

Warehouse

		W_1	W_2	W_3	Supply
Factory	F_1	16	20	12	200
	F_2	14	8	18	160
	F_3	26	24	16	90
	Demand	180	120	150	450

Determine the optimal distribution for this company in order to minimize its total shipping cost. (5)

b) Return distribution of two securities are given below:

Possible Rates of Return		Associated Probability
X	Y	$P_{xj} = p_{yj} = P_j$
0.16	0.14	0.33
0.12	0.08	0.25
0.08	0.05	0.17
0.11	0.09	0.25

Find which security is more risky in the Markowitz sense. (5)

6. a) Formulate the model for which the reproductive function of the cancer cells in the tumor surface is given by $\phi(c) = \dfrac{3-2c}{2(1-2)c}$; $c \neq \dfrac{1}{2}$ together with initial conditions $c = 20\times 10^5$ at $t = 0$. Also find the density of the cancer cells in the tumour's surface area at $t = 20$ days. (5)

b) Do the stability analysis of the trivial equilibrium solution of the following competing species model:

$$\frac{\partial N_1}{\partial t} = a_1 N_1 - b_1 N_1 N_2 + D_1 \frac{\partial^2 N_1}{\partial x^2}$$

$$\frac{\partial N_2}{\partial t} = -d_1 N_2 + c_1 N_1 N_2 + D_2 \frac{\partial^2 N_2}{\partial x^2},\ 0 \leq x \leq L,$$

where D_1 and D_2 are diffusion coefficients of the two population densities N_1 and N_2, respectively. a_1 is the growth rate, b_1 is the predation rate, d_1 is the death rate and c_1 is the conversion rate. The initial boundary conditions are

$$N_i(x, 0) = f_i(x) > 0,\ 0 \leq x \leq L,\ i = 1, 2$$

$$N_i = \overline{N}_i \text{ at } x = 0 \text{ and } x = L\ \forall\ t,\ i = 1, 2$$

where $\overline{N}_i$ are the equilibrium solutions of the given system of equations. (5)

7. a) Consider the data showing observations on the quantity demanded of a certain commodity depending on commodity price and consumers' income:

Quantity demanded	Price (in ₹)	Income (in ₹)
100	5	1000
75	7	600
80	6	1200
70	6	500
50	8	300
65	7	400
90	5	1300
100	4	1100
110	3	1300
60	9	300

Find the multiple regression equation that best fits the data. (5)

b) Consider the budworm population dynamics governed by the equation

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - x,$$

where k, the carrying capacity, and r, the birth rate of the budworm population, are positive parameters. Find out the steady states and use the perturbation to do the stability analysis of the equation for $0 < r < 1$. (5)

8. a) A company has three factories F_1, F_2, F_3 and these factories supply to three markets M_1, M_2, M_3. The transportation costs from each factory to each market are given in the table. Capacities 'a_i' s' of the factories and market requirements 'b_j' s' are also shown in the table. Find the minimum transportation cost. (6)

	M_1	M_2	M_3	a_i
F_1	2	1	3	20
F_2	1	2	3	30
F_3	2	1	2	10
b_j	10	10	20	40/60

b) A simple model including the seasonal change that affects the growth rate of a population is given by $\frac{dx}{dt} = Cx(t)\cos t$ where C is a constant. If x_0 is the initial population, solve the equation and determine the maximum and minimum population. (4)

9. a) Ships arrive at a port at the rate of one in every 4 hours, with exponential distribution of inter-arrival times. The time a ship occupies a berth for unloading has exponential distribution with an average of 10 hours. If the average delay of ships waiting for berths is to be kept below 14 hours, how many berths should be provided at the port? (5)

b) The yearly fluctuations in the groundwater table are believed to be dependent on the annual rainfall and the volume of water pumped out from the basin. The data collected on these variables for four consecutive years is given below:

Water table (in cm)	Annual rainfall (in cm)	Groundwater volume pumped out (in cm^3)
10	3	7
9	4	8
7	5	9
4	7	7

Use the method of least squares to find a linear regression equation that best fits the data. (5)

10. a) Consider the discrete time population model given

$$N_{t+1} = \frac{rN_t}{1 + \left(\frac{N_t}{K}\right)^b}, \text{ for a population } N_t,$$

where K is the carrying capacity of the population, r is the intrinsic growth rate and b is a positive parameter. Determine the non-negative steady-state and discuss the linear stability of the model for $0 < r < 1$. Also find the first bifurcation value of a the parameter r. (5)

b) The deviation $g(t)$ of a patient's blood glucose concentration from its optimal concentration satisfies the differential equation

$$\frac{d^2g}{dt^2} + 3\alpha\frac{dg}{dt} + 16\alpha^2 g = 0,$$

for α being a positive constant, immediately after the patient fully absorbs a large amount of glucose. The time t is measured in minutes. Identify the type (over-damped, under-damped or critically-damped) of this differential equation. Find the condition on α for which the patient is normal. (5)