MMT-004

ASSIGNMENT BOOKLET

(Valid from 1st January, 2022 to 31st December, 2022)

M.Sc. (Mathematics with Applications in Computer Science)
Computer Graphics (MMT-004)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi,
New Delhi-110068
(2022)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective courses that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO :……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO. ………………………….…

STUDY CENTRE: …………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved..
6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2022. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by December 2022, then you need to get the assignment for the year 2023 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the exam form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment (MMT – 004)

Course Code: MMT-004
Assignment Code: MMT-004/TMA/2022
Maximum Marks: 100

1. State whether the following statements are True or False. Give reasons for your answers. (10)

   a) Intersection of a dense set and a nowhere dense set in a metric space is always a dense set.

   b) The function $f(x, y) = (x + e^y, \sin|xy|)$ is continuously differentiable on $\mathbf{R}^2$.

   c) The point $(0, 0, 0)$ is a stationary point for the following function
      $f : \mathbf{R}^3 \to \mathbf{R}$ defined by
      $f(x, y, z) = (y - x^2)(y - 2x^2) + z$

   d) The outer measure of the interval $[3, 5]$ is 5.

   e) The set of all non-zero real numbers with usual metric is connected.

2. a) Verify whether the metrics on $\mathbf{R}^2$ given by

   $$d_1(\mathbf{x}, \mathbf{y}) = \max\{|x_1 - x_2|, |y_1 - y_2|\}$$

   and

   $$d_2(\mathbf{x}, \mathbf{y}) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

   are equivalent or not. (4)

   b) Show that the projection map $p = \mathbf{R}^2 \to \mathbf{R}$ given by $p(x, y) = y$ is continuous. (3)

   c) Let $X = \mathbf{R}^2$ with discrete metric d. Then show that the sequence $\left\{\left(\frac{3}{n}, \frac{3}{n}\right)\right\}_{n=1}^{\infty}$ does not converge to $(0, 0)$ in (X, d). (3)

3. a) Let $f(x, y, z) = x^2 e^y$, $g(x, y, z) = y^2 e^{xz}$ and $\varphi = (f, g)$. Find $\varphi''(1, 2, 3)$. (4)

   b) Find the interior, boundary and closure of the subset A of $\mathbf{R}^2$, where $A = \{(x, y) \in \mathbf{R}^2 : x^2 + y^2 = 6\}$ with the usual metric. (3)

   c) Give an example of a connected set in $\mathbf{R}$ which is i) countable, ii) uncountable. Justify your choice of examples. (3)

4. a) Suppose that X is a complete metric space and A is a non-empty subset of X. Show that A is totally bounded if $\overline{A}$ is compact. Is the converse true? Justify your answer. (4)

   b) Show that arbitrary intersection of sets which are both compact and closed, is compact. (3)

c) Justify by an example that the image of a Cauchy sequence under a continuous map need not be Cauchy. (3)

5\. a) Verify implicit function theorem for the function $f(x, y) = y^2 - yx^2 - 2x^5$ in the neighbouhood of the point $(1, -1)$ and hence find f'. (3)

b) Show that the function $f : \mathbf{R}^4 \to \mathbf{R}^4$ defined by $f(x, y, z, w) = (x + 2y, x^2 - y^2, w^3, y + w)$ is locally invertible at the point $(0, 1, -1, 2)$. (3)

c) Check the function $f : \mathbf{R}^3 \to \mathbf{R}$, defined by $f(x, y, z) = x^2y + y^2z + z^2 - 2x$ for local extrema. (4)

6\. a) Show that every sequence in a compact metric space has a subsequence which is Cauchy. (3)

b) Obtain the second Taylor's series expansion for the function f given by

$f(x, y) = x^2y + xe^y$ at $(1,0)$. (4)

c) Find the outer measure of the set

$$E = \{x \in \mathbf{R} : \sin x = 0\} \cup \left[\frac{1}{2}, 1\right].$$ (3)

7\. a) Obtain the second Taylor series expansion for the function $f : \mathbf{R}^2 \to \mathbf{R}$ given by $f(x, y) = x^2y + xe^4$ at $(1, 8)$. (4)

b) Show by an example that the condition that the Jacobian Vanishes at a point is not necessary for a function to be locally invertible at that point. (4)

c) Is the set of irrational numbers measurable? Justify your answer. (2)

8\. a) Let $C[a, b]$ denote the set of measurable functions which are continuous on $[a, b]$. Show that $C[a, b] \subset L^1[a, b]$. (3)

b) Check the differentially of the following functions F at the indicated points wherever exists.

i) $F : \mathbf{R}^2 \to \mathbf{R}$ defined by $F(x, y) = |x| + |y|$ **at (1, 0).**

ii) $F : \mathbf{R} \to \mathbf{R}^2$ defined by $F(x) = (f_1(x), f_2(x))$ where $f_1(x) = x$ and

$$f_2(x) = \begin{cases} x \sin 1/x, & (x \neq 0) \\ 0, & (x = 0) \end{cases}$$ **at 0**

iii) $F : \mathbf{R}^4 \to \mathbf{R}^3$ defined by $F(x, y, z, w) = (x^2 + y^2 + zw^2, x^2y, xyz)$ **at (1, 2, −1, 2).** (4)

c) Does the sequence $\{f_n\}$ where $f_n = \chi_{[n,n+1]}, n = 1, 2...$ satisfy the conditions of monotone convergence theorem? Does the conclusion of the theorem hold good for this sequence? Justify (3)

9. a) Compute the Fourier Series of the function g given by $g(t) = \begin{cases} -2, & -\pi < t < 0 \\ 2, & 0 < t < \pi \end{cases}$. (2)

b) Let f and g be the functions given by

$$f(t) = \begin{cases} \sqrt{t}, & \text{if } 0 < t < 1 \\ 0, & \text{if } t \le 0 \text{ or } t \ge 1 \end{cases}$$

$$g(t) = \begin{cases} \sqrt{1-t}, & \text{if } 0 < t < 1 \\ 0, & \text{if } t \le 0 \text{ or } t \ge 1 \end{cases}$$

Find $f * g$ and $g * f$ and verify that they are the same. (5)

c) Check whether the following function is Lebesgue measurable or not. (3)

$$f(x) = \begin{cases} x^2, & \text{if } x \in Q \\ -x^2, & \text{if } x \notin Q \end{cases}$$

10. a) Find the outer measure of the following sets. (3)

i) $A = [-1, 1] \cup [x \in \mathbf{R} : x^3 - 8 = 0\}$

ii) $B = (0, 1) \cup \left(\frac{1}{2}, 2\right) \cup \left(\frac{3}{2}, 4\right)$.

b) Use the method of Lagrange's multiplier to find the point on the line of intersection of the planes $x - y = 2$ and $x - 2z = 4$ that is closest to the origin. (4)

c) Consider the function $f : \mathbf{R}^4 \to \mathbf{R}^2$ given by $f(x, y, z, w) = (x^2 - y^2, z^2 - w^2)$. Check whether the second order partial derivatives of this function exists at $(1, 0, 2, -1)$. (3)