

MMTE-002

ASSIGNMENT BOOKLET

(Valid from 1st January, 2021 to 31st December, 2021)

M.Sc. (Mathematics with Applications in Computer Science)

DESIGN AND ANALYSIS OF ALGORITHMS



**School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi
(2021)**

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 percent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

- 1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO. :

NAME :

ADDRESS :

.....

.....

COURSE CODE :

COURSE TITLE :

STUDY CENTRE : **DATE**

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

- 2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
- 3) Leave a 4 cm margin on the left, top and bottom of your answer sheet.
- 4) Your answers should be precise.
- 5) While solving problems, clearly indicate which part of which question is being solved.
- 6) This assignment is to be submitted to the Programme Centre as per the schedule made by the Programme Centre. Answer sheets received after the due date shall not be accepted.
- 7) This assignment is valid only up to December, 2021. If you fail in this assignment or fail to submit it by December, 2021, then you need to get the assignment for the year 2022 and submit it as per the instructions given in the Programme Guide.
- 8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So, solve it and **submit it to your study centre at the earliest**.
- 9) We **strongly** suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment

Course Code: MMTE-002

Assignment Code: MMTE-002/TMA/2021

Maximum Marks: 100

Instructions: In this assignment, 'book' refers to the textbook 'Introduction to Algorithms' by Cormen et. al. In this assignment we follow the notations, symbols and conventions followed in this book.

- 1) a) Write an algorithm to find the first two largest numbers in an in array of integers. For example, given the input $\langle 2, 3, 7, -4, 5, 1 \rangle$, the algorithm should output 7 5. State precisely a loop invariant for you algorithm. Prove that your loop invariant holds and hence conclude that your algorithm works. (5)

Analyse the algorithm to find the upper bound for run time of the above algorithm.

- 2) a) With the help of an example, explain the following:
i) Algorithm.
ii) Input and output for an algorithm.
iii) Running time of an algorithm. (5)
- b) Using Fig. 7.1 in page 147 (page 172 in the third edition) of the book as the model, illustrate the operation of PARTITION on the array $A = \{3, 10, 8, 5, 12, 9, 6, 13, 4, 2, 7\}$ (5)
- 3) a) Show step by step how you will construct a binary search tree by inserting keys in the order 12, 6, 4, 7, 8, 10, 7, 5, 13, 16, 11, starting from an empty binary tree. Explain step by step how you will delete the node with the key 13 from the binary tree. (5)
- b) Using Fig. 6.4 in page 137 (page 161 in third edition) of the book as a model, illustrate the operation of HEAPSORT on the array $A = \{3, 4, 2, 3, 1, 6, 5, 7, 5\}$. (5)
- 4) a) Show the results of inserting the keys

$C, R, A, U, J, S, F, T, O, P, M, L, N, W, Q$

in order into an empty B-tree with minimum degree 2. Only draw the configurations of the tree just before some node must split, and also draw the final configuration. (5)

- b) Suppose the CONNECTED-COMPONENTS is run on the undirected graph $G = (V, E)$, where $V = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and the edges in $E(V) = \{e_1 = (1, 3), e_2 = (2, 4), e_3 = (3, 6), e_4 = (7, 8), e_5 = (2, 5), e_6 = (3, 9), e_7 = (4, 7), e_8 = (3, 8)\}$ are processed in the order $\{e_1, e_2, e_3, e_4, e_6, e_5, e_7, e_8\}$. List the vertices in each connected component after each iteration of lines 3–5 in the CONNECTED-COMPONENTS. (5)
- 5) a) Show step by step how the merge procedure of merge sort will merge the arrays 1, 3, 4, 7, 9, 11, 13, 14 and 2, 5, 6, 8, 10, 12 (3)
- b) For the following set of points, describe how the CLOSEST-PAIR algorithm finds a closest pair of points: (3)

$(1, 2), (2, -1), (3, -3), (3, 2), (-3, 4), (-1, 2), (2, 3), (1, 1), (-5, -2)$

- c) Find an optimal parenthesisation of a matrix chain product whose sequence of dimensions is $(4, 8, 7, 2, 3)$. (4)

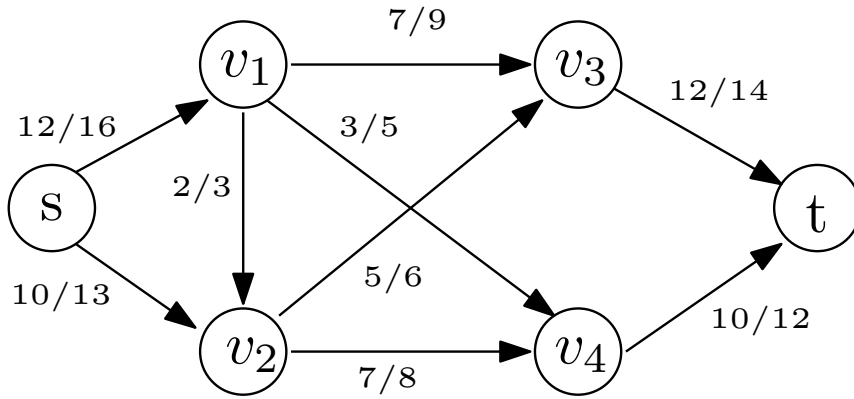


Figure 1

- 6) a) Figure 1 depicts a flow network and a flow. Find an augmenting path and the residual network. (5)
- b) Determine an LCS of $(1, 1, 1, 1, 0, 1, 1, 0, 1, 0)$ and $(1, 1, 1, 0, 0, 1, 1, 0, 1, 0)$. (5)
- 7) a) Show the d and π values that result from running depth-first search on the graph given in Fig. 2 using vertex 4 as the source. (7)

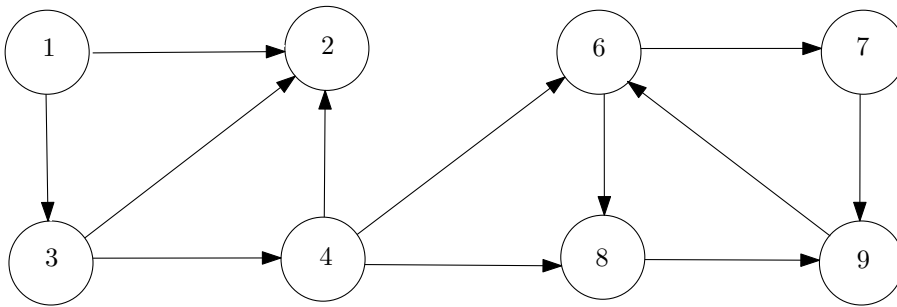


Figure 2: Figure for exercise 7(a).

- b) Use Prim's algorithm to find a minimal spanning tree in the graph given in Fig. 3, starting with the vertex labelled 2. (7)

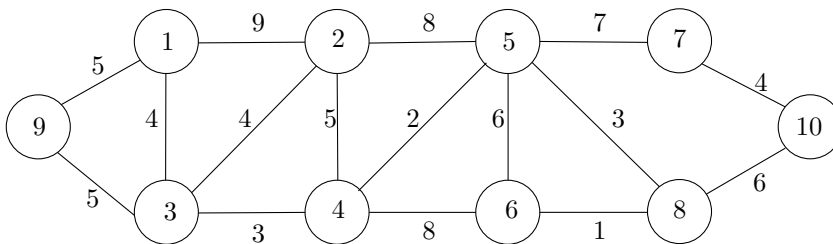


Figure 3: Figure for exercise 7(b)

- c) Use Dijkstra's algorithm to find the shortest paths in the graph given in Fig. 4 with a as the source vertex. (6)

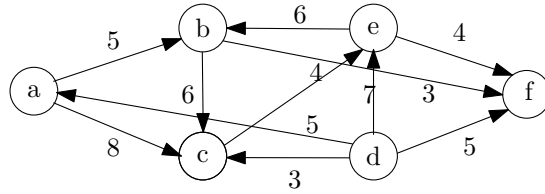


Figure 4: Figure for exercise 6(c).

- 8) a) Show the comparisons the naive string matcher makes for the pattern $P = 0110$ with 01101000110101100 . (4)
- b) When working modulo $q = 13$, how many spurious hits does the Rabin-Karp matcher encounter in the text $T = 294872235748$ when looking for the pattern 22? (3)
- c) Compute the values (d, x, y) that the call `EXTENDED-EUCLID(19177,26797)` returns. (3)
- 9) a) Find all the solutions of the equation $7x \equiv 28 \pmod{119}$. (3)
- b) Let $\{(-2, -6), (-1, 5), (0, 4), (1, 3)\}$ and $\{(-2, -18), (-1, -7), (0, -4), (1, -3)\}$ be the point-value representations of two polynomials $f(x)$ and $g(x)$. Find the point-value representation of $h(x) = f(x) + g(x)$. From the point value representation of $h(x)$ find the coefficient representation of $h(x)$. (4)
- c) Compute the DFT of the vector $(-3, 1, 1, -1)$. (3)