MMT-009

ASSIGNMENT BOOKLET

M.Sc. (Mathematics with Applications in Computer Science)
Mathematical Modelling (MMT-009)
(January 2021– December 2021)

School of Sciences
Indira Gandhi National Open University
Maidan Garhi,
New Delhi-110068

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective courses that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO :……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO. ………………………….…

STUDY CENTRE: ………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved..

6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto December, 2021. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by December 2021, then you need to get the assignment for the year 2022 and submit it as per the instructions given in the programme guide.

We wish you good luck.

Assignment (MMT-009)

Course Code: MMT-009
Assignment Code: MMT-009/TMA/2021
Maximum Marks: 100

1. Do the stability analysis of the following competing species system of equations with diffusion and advection

$$\frac{\partial N_1}{\partial t} = a_1 N_1 - b_1 N_1 N_2 + D_1 \frac{\partial^2 N_1}{\partial x^2} - v_1 \frac{\partial N_1}{\partial x}$$

$$\frac{\partial N_2}{\partial t} = -d_1 N_2 + C_1 N_1 N_2 + D_2 \frac{\partial^2 N_2}{\partial x^2} - V_2 \frac{\partial N_2}{\partial x}, \quad 0 \le x \le L$$

where V_1 and V_2 are advection velocities in x direction of the two populations with densities N_1 and N_2 respectively. a_1 is the growth rate, b_1 is the predation rate, d_1 is the death rate, C_1 is the conversion rate. D_1 and D_2 are diffusion coefficients. The Initial and boundary conditions are:

$$N_i(x,0) = f_i(x) > 0,\ 0 \le x \le L,\ i = 1, 2\Lambda$$

$$N_i = \overline{N}_i \text{ at } x = 0 \text{ and } x = L\ \forall\ t, i = 1, 2\Lambda$$

where $\overline{N}_i$ are the equilibrium solutions of the given system of equations.
Interpret the solution obtained and also write the limitations of the model. (10)

2. a) Companies located on the banks of a river and producing chemicals dispose their waste by indis criminately dumping it into the river, causing high levels of pollution. Local authorities passed new legislation with very high fines if the pollution in the river exceeds certain specified concentration limits. **State, giving reasons,** the type of modelling you will use to find a policy for discharging the waste so as to ensure that the concentration level never exceeds the specified limits. Also state four essentials and two non-essentials for the problem. (6)

   b) Characterise the following as discrete or continuous giving reasons for your answer.

      i) Effects of radiation treatment on a tumour when applied for short period of time but at regular intervals.

      ii) Effects of chemotherapy drugs on a tumour when introduced into a patient for a given duration of time. (2)

   c) Bring out the difference between linear and non-linear models giving examples. Examples should be different from those given in your blocks. (2)

3. a) When $\rho_{12} = +1$, the standard deviation of a two-security portfolio P is equal to the weighted average of the standard deviations of its component securities. Is this statement true? Give reasons for your answer. (2)

   b) Explain how a portfolio along the efficient frontier is better than other portfolios in the feasible set? (3)

   c) Which one of the following portfolios cannot lie on the efficient frontier as described by Markowitz?

Portfolio	**Expected Return**	**Standard Deviation**

W	9%	21%
X	5%	7%
Y	15%	36%
Z	12%	15%

(5)

4. Assume that the return distribution on the two securities X and Y be as given below:

	Market 1	**Market 2**	**Market 3**
Probability	0.2	0.5	0.3
Security X	-20%	18%	50%
Security Y	-15%	20%	10%

Find σ_{XY} and ρ_{XY} . (10)

5. a) The data showing the time required for a merchandiser to stock a grocery store shelf with a soft drink product as well as the number of cases of product stocked is given below.

Time	**Cases Stocked**
10.15	25
2.96	6
3.00	8
6.88	17
0.28	2
5.06	13
9.14	23
11.86	30
11.69	28
6.04	14
7.57	19
1.74	4
9.38	24
0.16	1
1.84	5

i) Draw a scatter plot of the data. (1)

ii) Fit a linear regression model to the data. (2½)

iii) Obtain the residual for the fitted line. (1½)

b) Consider the data showing observations on the quantity demanded of a certain commodity depending on commodity price and consumers income.

Quantity demanded	**Price (in Rs.)**	**Income (in Rs.)**
100	5	1000
75	7	600
80	6	1200

70	6	500
50	8	300
65	7	400
90	5	1300
100	4	1100
110	3	1300
60	9	300

i) Find a linear regression equation that best fit the data. (3)

ii) Obtain the coefficient of multiple determination for the data. (2)

6. a) In a species of animals a constant fraction of the population $\alpha = 5.3$ are born each breeding season and a constant fraction $\beta = 4.97$ die. Formulate a difference equation for the population and find out the number of individuals after fifteen seasons given that the initial number is 987. Find the closed form solution of the formulated difference equation. If the growth rate of the population is represented by r then interpret the solution obtained when i) $r > 0$ and ii) $r < 0$. (6)

b) A model for insect populations leads to the difference equations

$$N_{k+1} = \frac{\lambda N_k}{1 + a\,N_k}$$

where λ and a are positive constants.

i) Write the equation in the form $N_{k+1} = N_k + R(N_k)N_k$ and hence identify the growth rate.

ii) Plot the graph of $R(N_k)$ as a function of N_k.

iii) Express the intrinsic growth rate r and the carrying capacity K, for this model, in terms of the parameters, a and λ.

iv) Find the steady-state solution of this model and analyse the solution. (4)

7. a) What are residual plots and box plots? (3)

b) Do the stability analysis of the following model which is formulated to study the effect of toxicant on one competing species where the environmental toxicant concentration is being taken to change w.r.t. time.

$$\frac{dN_1}{dt} = r_1 N_1 - \alpha_1 N_1 N_2 - d_1 C_0 N_1$$

$$\frac{dN_2}{dt} = r_2 N_2 - \alpha_2 N_1 N_2$$

$$\frac{dC_0}{dt} = k_1 P - g_1 C_0 - m_1 C_0$$

$$\frac{dP}{dt} = Q - hp - kPN_1 + gC_0 N_1.$$

along with the initial conditions.

$$N_1(0) = N_{10}, N_2(0) = N_{20}, C_0(0) = 0,\ P(0) = P_0 > 0$$

Here,

N_1 (t) = Density of prey population

$N_2(t)$ = Density of predator population

$C_0(t)$ = Concentration of the toxicant in the individuals of the prey population

P = Constant environmental toxicant concentration.

α_1, α_2 are the predation rates, r_1, r_2 are the growth rates or birth rates, d_1 is the death rate due to C_0, m_1 is the depuration rate, Q, h, k, g are positive rate constants. (7)

8. a) Formulate the model for which the reproductive function of the cancer cells in the tumour surface is given by $\phi(c) = \frac{c-1}{1-2c}; \ c \neq \frac{1}{2}$ together with initial conditions $c = 20 \times 10^5$ at $t = 0$. Also find the density of the cancer cells in the tumour's surface area at t = 45 days. (5)

b) The deviation g (t) of a patient's blood glucose concentration from its optimal concentration satisfies the differential equation $4\frac{d^2g}{dt^2} + 8\alpha\frac{dg}{dt} + (2\alpha)^2 g = 0$, α a constant, immediately after he fully absorbs a large amount of glucose. The time t is measured in minutes. Find the value of α for which the patient is normal. (3)

c) Solve the model for the detection of diabetes for critically damped case. (2)

9. a) A company had three factories that supply to three markets. The transportation costs from each factory to each market are given in the table. Capacities of the factories and market requirements are shown. Find the minimum transportation cost.

	M_1	M_2	M_3	a_i
F_1	2	1	3	20
F_2	1	2	3	30
F_3	2	1	2	10
b_j	10	10	20	40/60

(6)

b) For a multi-channel queueing system with $\lambda = 12/\text{hours}, \mu = 5/\text{hours}, c = 3, p_0 = 0.056$, calculate

i) The average time a customer is in the system

ii) The average number of customers in the system

iii) whether any time would be saved for customers if the three-channel system with the service rate of 5 per hour is replaced by a single-channel system with an average service rate of 15 per hour? (4)

10. The owner of a readymade garments store sells two types of shirts-Zee-shirts and Button-down shirts. He makes a profit of Rs.3 and Rs.12 per shirt on Zee-shirts and Button-down shirt, respectively. He has two tailors, A and B at his disposal to stitch the shirts. Tailors A and B can devote at the most 7 hours and 15 hours per day, respectively. Both these shirts are to be stitched by both the tailors. Tailors A and B spend 2 hours and 5 hours, respectively in stitching one Zee-shirts, and 4 hours and 3 hours, respectively in stitching a Button-down shirt. How many shirts of both types should be stitched in order to maximize daily profit?

i) Formulate and solve this problem as an LP problem.

ii) If the optimal solution is not integer-valued, use Gomory technique to derive the optimal integer solution. (10)