MMT-007

ASSIGNMENT BOOKLET

(Valid from 1st January, 2021 to 31st December, 2021)

M.Sc. (Mathematics with Applications in Computer Science)

DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

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THE PEOPLE'S UNIVERSITY

School of Sciences
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(2021)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 20 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………….

COURSE TITLE : …………………………….

ASSIGNMENT NO.: ………………………….…

STUDY CENTRE: ………………………..….. DATE: ……………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) While solving problems, clearly indicate which part of which question is being solved.
6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

   We strongly suggest that you retain a copy of your answer sheets.
7) This assignment is valid only upto December, 2021. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by December, 2021, then you need to get the assignment for the year 2022 and submit it as per the instructions given in the programme guide.
8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: MMT-007
Assignment Code: MMT-007/TMA/2021
Maximum Marks: 100

1. a) Show that $J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \cdots) = 1$. (3)

   b) If $f(x) = 0, \quad -1 < x \le 0$
   $\quad\quad = x, \quad 0 < x < 1$

   Show that

   $$f(x) = \frac{1}{4}P_0(x) + \frac{1}{2}P_1(x) + \frac{5}{16}P_2(x) - \frac{3}{32}P_4(x) + \cdots$$

   where $P_n(x)$ is a Legendre polynomial of degree n. (3)

   c) Show that

   $$H_n(x) = \frac{(-i)^n 2^n e^{x^2}}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-t^2+2ixt}\, t^n dt$$

   where $H_n(x)$ is the Hermite polynomial of degree n. (4)

2. a) Find the power series solution about $x = 0$ of the differential equation
   $9x(1-x)\ y'' - 12y' + 4y = 0$. (5)

   b) Construct Green's function for the boundary value problem

   $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 0, \quad 0 < x < \pi/2$$

   $y(0) = 0, \ \ y(\pi/2) = 0$. (5)

3. a) Show that

   $$J_{5/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{1}{x^2}(3 - x^2)\ \sin x - \frac{3}{x}\cos x\right).$$ (2)

   b) For the function $f(x) = x - x^3,\ 0 < x < 1$, obtain the Bessel series of the form
   $f(x) = a_1 J_1(\lambda_1 x) + a_2 J_1(\lambda_2 x) + \cdots$
   where λ_n's are the positive zeros of $J_1(x)$. (4)

   c) Find the cosine and sine integral representations of the function
   $f(x) = xe^{-2x},\ x > 0$. (4)

4. a) Apply the convolution theorem to evaluate $\mathcal{L}^{-1}\left[\frac{s}{(s^2 + a^2)^2}\right]$. (4)

   b) Using the Fourier transform method, solve the two-dimensional Laplace equation

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}=0,\quad 0<x<\pi,\ 0<y<y_0$$

subject to $v\,(0,\ y)=0$, $v\,(\pi,\ y)=1$

$v_y(x,\ 0)=0$, $v\,(x,\ y_0)=1$ (6)

5. a) Show that $\mathcal{L}^{-1}\left[\frac{e^{-1/s}}{\sqrt{s}}\right]=\frac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$. (5)

b) Solve the following IBVP using the Laplace transform technique:

$u_{tt}=u_{xx}$, $\quad 0<x<1,\quad t>0$

$u(0,t)=u(1,\ t)=0,\qquad t>0$

$u(x,\ 0)=\sin\pi x,\ u_t(x,\ 0)=-\sin\pi x,\quad 0<x<1$ (5)

6. a) Solve the IVP

$y'=y+\cos y,\quad y(1)=0$

upto $x=2$ using the Milne-Simpson's predictor-corrector method

$$P: y_{n+1}^{(p)}=y_{n-3}+\frac{4h}{3}\,[2f_n-f_{n-1}+2f_{n-2}]$$

$$C: y_{n+1}^{(c)}=y_{n-1}+\frac{h}{3}\,[f_{n-1}+4f_n+f(x_{n+1},\ y_{n+1}^{(p)})]$$

with the step length $h=0.2$. Calculate the starting values using Euler's method with the same h. (6)

b) Investigate the stability of the method

$$\delta_t^2\ u_i^n=\frac{r^2}{2}\,\delta_x^2\left[u_i^{n+1}+u_i^{n-1}\right],\ r=\frac{k}{h}$$

for approximating the wave equation $u_{tt}=u_{xx}$. (4)

7. a) Find an approximate value of $y(0.6)$ for initial value problem $y'=x-y^2$, $y(0)=1$ using multistep method $y_{i+1}=y_i+\frac{h}{2}(3f_i-f_{i-1})$ with step length $h=0.2$. Compute the starting value using Taylor series second order method with same steplength. (3)

b) Solve the boundary value problem

$y''-5y'+4y=0$

$y(0)-y'(0)=-1,\ y(1)+y'(1)=1$

using second order finite differences for y' and y'', with $h=1/2$. (7)

8. a) Using standard five point formula, solve the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ in R

subject to the boundary conditions

$u(x,\ 0)=2x,\quad u(0,\ y)=-y$

$u(x,\ 1)=2x-1,\ u(1,\ y)=2-y$

with $h=k=1/3$ where R is the square $0\le x\le 1,\ 0\le y\le 1$. (5)

b) Find the solution of the initial boundary value problem

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1$$

$$u(x,0) = \sin(\pi x),\ 0 \le x \le 1,\ \ \frac{\partial u}{\partial t}(x,\ 0) = 0,\ \ 0 \le x \le 1$$

$$u(0,\ t) = u(1,\ t) = 0,\ t > 0$$

by using second order explicit method with $h = \frac{1}{4}$, $r = \frac{1}{3}$. Integrate for two time steps. (5)

9. a) Using the Schmidt method, with $\lambda = \frac{1}{6}$, find the solution of the initial value problem

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

subject to the conditions

$$u(0,\ t) = 0 = u(1,\ t)$$

and $$u(x,\ 0) = \begin{cases} 2x & \text{for } x \in [0,\ 1/2] \\ 2(1-x) & \text{for } x \in [1/2,\ 1] \end{cases}$$

with $h = 0.2$. (3)

b) Using the five-point formula and assuming the uniform step length $h = \frac{1}{3}$ along the axes, find the solution of $\nabla^2 u = x^2 + y^2$ in R where, R is the triangle $0 \le x \le 1$, $0 \le y \le 1,\ 0 \le x + y \le 1$. On the boundary of the triangle $u(x,\ y) = x^2 - y^2$. (7)

10. State whether the following statements are true or false. Justify your answer with the help of a short proof or a counter example. (10)

i) $\cos z = J_0(z) + 2\sum_{n=1}^{\infty} (-1)^m\ J_{2m}(z)$

ii) The initial value problem $y'(x) = \begin{cases} \dfrac{2y}{x}, & x > 0 : y(0) = 0 \\ 0, & x = 0 \end{cases}$ has a unique solution in any closed rectangle containing origin.

iii) If H_n is a Hermite polynomial of degree n, then $H_n' = 4n(n-1)H_{n-2}$.

iv) The interval of absolute stability for 3rd order Taylor series method for IVP $y' = \lambda y,\ y(x_0) = y_0$ is $]-2.78,\ 0[$.

v) For $\lambda = \frac{1}{6}$, the Schmidt method for solving parabolic equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ in a region $R(a \le x \le b)$ and $0 \le t \le T$ is of order $0(k^2 + h^4)$.