MMT-005

ASSIGNMENT BOOKLET

(Valid from 1st January, 2021 to 31st December, 2021)

M.Sc.(Mathematics with Applications in Computer Science)

COMPLEX ANALYSIS

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi
(2021)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 percent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

	ROLL NO. : ....
	NAME : ....
	ADDRESS : ....
	....
	....
COURSE CODE :	....
COURSE TITLE :	
STUDY CENTRE :	**DATE** ....

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **Solve the assignment on your own. Don't copy from your fellow students or from the internet. If you are found guilty of copying, your assignment will be disqualified and you will have to submit the assignment for the next session.**

7) This assignment is to be submitted to the Programme Centre as per the schedule made by the Programme Centre. Answer sheets received after the due date shall not be accepted.

8) This assignment is valid only up to December, 2021. If you fail in this assignment or fail to submit it by December, 2021, then you need to get the fresh assignment for the year 2022 and submit it as per the instructions given in the Programme Guide.

9) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So, solve it and **submit it to your study centre at the earliest.**

10) We **strongly** suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment

Course Code: MMT-005
Assignment Code: MMT-005/TMA/2021
Maximum Marks: 100

1) Which of the following statements are true and which are false? Give reasons for your answer.

i) The Set $S = \{z \in \mathbb{C} : |z + ia| < |z - a| \text{ for some } a > 0\}$ is a domain.

ii) For any positive integer n, $|\operatorname{Im} z^n| \leq n\,|\operatorname{Im} z|\,|z|^{n-1}$.

iii) The transformation $w = 2^{-1}[z + \alpha^2 z^{-1}]$ $(\alpha \in \mathbb{R})$, maps the circle $|z| = r$ $(r \neq \alpha)$ into an ellipse in the w-plane.

iv) An entire function f such that $f(x + iy) = u(x) + iv(y)$ must be of the form $f(z) = \alpha z + \beta$ with $\alpha \in \mathbb{R}$, $\beta \in \mathbb{C}$.

v) The function f defined by

$$f(z) = \begin{cases} 0 & \text{if} z = 0 \\ \exp\left(-1/z^4\right) & \text{if} z \neq 0 \end{cases}$$

is continuous at the origin.

vi) There does not exist an analytic function f in a neighbourhood of 0 whose square is z.

vii) If f is a complex-valued continuous function on $\mathbb{C}$, then $I = \int_{|z|=1} \frac{f(z)-f(1/z)}{z}\,dz = 0$.

viii) If $p(z)$ is a polynomial of degree n in z with complex coefficient, then

$$I = \int_{|z|=1} p(\bar{z})\,dz = 2\pi i p'(0).$$

ix) The set S of all linear fractional transformations satisfy the commutative property with respect to composition.

x) Suppose f is an entire function which takes real z into real and purely imaginary into purely imaginary. Then f is an even function. (20)

2) a) Show that the function $f(z) = \dfrac{z}{(1-z)^2}$ is one-to-one in the unit disc

$D(0,\ 1) = \{z \in \mathbb{C} : |z| < 1\}$.

Also find the range of $f(z)$. (4)

b) Show that the function f defined by

$f(z) = |\operatorname{Re} z \operatorname{Im} z|^{1/2}$

satisfy the C-R equations at the origin, but is not differentiable at this point. (3)

c) Find the most general cubic form

$u(x, y) = ax^3 + bx^2y + cxy^2 + dy^3 (a, b, c, d \text{ real})$

which satisfies the Laplace's equation and also determine an analytic function which has u as its real part. (3)

3) a) Sketch the families of level curves of the component functions u and v when $f(z) = \frac{z-1}{z+1}$. (3)

b) Find the principal branch of the logarithm of $z = -3$ (2)

c) Sketch the following curve:

$$z(t) = \begin{cases} t + i(1+t^2), & -2 \le t \le 1 \\ 2 - t + i[1 + (t-2)^2], & 1 < t \le 4 \end{cases}$$

Check whether it is

i) closed

ii) simple

iii) smooth

iv) piecewise smooth (5)

4) a) If f is an entire function such that $\int_0^{2\pi} \left| f\left(re^{i\theta}\right) \right| d\theta \le r^\alpha$ for some fixed $\alpha > 0$ and for all $r > 0$, then show that $f(z) = 0$ in C. (4)

b) Show that the Taylor series $\sum_{n=1}^{\infty} n^{-1}(z-3)^n$ converges to $-\operatorname{Log}(4-z)$ for $|z-3| < 1$.(4)

c) Show that $I = \int_{|z|=r} |z - r|\ |dz| = 8r^2$. (2)

5) a) If f is entire and $f(z) = f(-z)$ for all z, then there exists an entire function g such that $f(z) = g\left(z^2\right)\ \forall z \in \mathbb{C}$. (3)

b) Evaluate the integral

$$I = \int_C \operatorname{Re}\left(z^2\right)\ dz$$

from 0 to $2 + 4i$ along the

i) line segment joining the points $(0,\ 0)$ and $(2,\ 4)$

ii) parabola $y = x^2$. (7)

6) a) Find the Taylor expansion about 0 for $f(z) = \dfrac{z-1}{z^2 - z - 1}$ and determine its radius of convergence. (3)

b) Write the Maclaurin series representation of the function $f(z) = \sin z^2$ and hence show that $f^{(4n)}(0) = 0$ and $f^{(2n+1)}(0) = 0$ $(n = 0,\ 1,\ 2,\ \ldots)$. (3)

c) Write the two Laurent series in powers of z that represent the function $f(z) = \frac{1}{z(1+z^2)}$ in certain domain and specify the domains. (4)

7) a) Suppose that f and g are analytic at a, $g^{(k)}(a) = 0 = f^{(k)}(a)$ for $k = 0,\ 1,\ 2,\ \ldots,\ n-1$ but both are not identically equal to zero. If $g^{(n)}(a) \ne 0$ show that

$$\lim_{z \to a} \frac{f(z)}{g(z)} = \frac{f^{(n)}(a)}{g^{(n)}(a)}$$

provided the limit on the right exists. (3)

b) Show that $\displaystyle\int_C \frac{dz}{(z^2-1)^2 + 3} = \frac{\pi}{2\sqrt{2}}$ where C is the unit circle in the positve sense. (4)

c) Evaluate the integral of the function $f(z) = \frac{z^5}{1-z^3}$ around the circle $|z| = 2$ in the positive sense. (3)

8) a) Find the image of the semi-infinite strip $x > 0,\ 0 < y < 1$ when $w = i/z$. Sketch the strip and its image. (5)

b) Show that there is only one linear fractional transformation that maps three given distinct points $z_1,\ z_2$ and z_3 in the extended z plane onto three specified distinct points $w_1,\ w_2$ and w_3 in the extended w plane. (5)

9) Evaluate the following integrals

a) $$\int_0^{\infty} \frac{x^2+2}{(x^2+1)\,(x^2+4)}\,dx.$$ (5)

b) $$\int_{-\infty}^{\infty} \frac{\sin^2 2x}{1+x^2}\,dx.$$ (5)