MPH-011

ASSIGNMENT BOOKLET

M.Sc. (Physics) Programme
(MSCPH)

STATISTICAL MECHANICS

Valid from 1st July, 2024 to 30th June, 2025

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2024-25)

Dear Student,

Please read the section on assignments in the Programme Guide for M.Sc. (Physics). A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet. The total marks for this assignment is 100, of which 40 marks are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ENROLMENT NO.: ……

NAME: ……

ADDRESS: ……

COURSE CODE:…….

COURSE TITLE: ……

ASSIGNMENT CODE: ……..

STUDY CENTRE: …….. **DATE:** …….

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) **Submit the assignment answer sheets within the due date.**

6) The assignment answer sheets are to be submitted to your Study Centre as per the schedule. **Answer sheets received after the due date shall not be accepted. We strongly suggest that you retain a copy of your answer sheets.**

7) This assignment is **valid from 1st July 2024 to 30th June 2025**. If you have failed in this assignment or fail to submit it by June 30, 2025, then you need to get the assignment for the year 2025-26, and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the examination form for this course** until you have submitted this assignment. For any queries, please contact: drsgupta@ignou.ac.in and mbnewmai@ignou.ac.in

We wish you good luck.

Tutor Marked Assignment
STATISTICAL MECHANICS

Course Code: MPH-011
Assignment Code: MPH-011/TMA/2024-25
Max. Marks: 100

Note: Attempt all questions. The marks for each question are indicated against it.

PART A

1. a) Calculate the mean, and the variance of the following probability density functions:

   i) Uniform $p(x) = \frac{1}{2a}, -a < x < a$, and $p(x) = 0$ elsewhere.

   ii) Rayleigh distribution $p(x) = \frac{x}{a^2} e^{-\frac{a^2}{2a^2}}$. (10)

   b) Show that for any distribution where N is large, the distribution tends to a Gaussian or a normal distribution. (10)

2. a) Consider a single classical 1-D non-relativistic harmonic oscillator. Obtain $x(t)$ and $p(t)$ as a function of time t. Assume that initially at $t = 0$, the oscillator is at the maximum displacement x_0 and $p(0) = 0$. Plot its phase space trajectory for fixed total energy E. (5)

   b) What is Gibb's paradox? Derive Sackur-Tetrode equation of entropy for a classical, ideal gas. (5)

   c) Obtain an expression of probability of finding the system in the microstate k corresponding to energy E_k in canonical ensemble. Also, write the expression of canonical partition function. (5)

   d) Calculate relative fluctuations in the energy of the system about the mean value in canonical ensemble. (5)

   e) Consider a gas in lattice-gas model, in which gas contained in volume V is assumed to be divided into fixed small cubic cells of volume V_0 such that each of the (cell) can contain at most one atom. The system behaves as a set of distinguishable and independent cells in contact with a reservoir at temperature T and chemical potential μ. Obtain the expressions of canonical partition function and average number of particles $\langle N \rangle$. (5)

3. Derive quantum Liouville equation. Write this equation for the Steady state. (5)

PART B

4. a) Using the result $\hat{\rho} = \frac{e^{-\beta \hat{H}}}{Tr\left(e^{-\beta \hat{H}}\right)}$, show that the expression of density matrix $(\hat{\rho})$ of a free particle in box of volume V in the canonical ensemble in the coordinate representation is given by

$$Tr(e^{-\beta \hat{H}}) = V\left(\frac{m}{2\pi\beta\hbar^2}\right)^{3/2} \quad (5)$$

b) Obtain Fermi-Dirac distribution function. Draw it for $T = 0K$ and $T > 0K$. What is its physical significance? (5)

c) Explain Pauli's paramagnetism. Show that at $T = 0$ Pauli's paramagnetic susceptibility is given by

$$\chi_0 = \frac{3}{2} n \mu^{*2} / \varepsilon_F$$

where μ^* is the intrinsic magnetic moment of the particle of mass m. (5)

d) How many photons are present in 1cm^3 of radiation at 727°C? What is their average energy?

$$\left(\text{Take : } \quad \int_0^\infty \frac{x^2 dx}{e^x - 1} = 2.405 \right). \tag{5}$$

5. a) Derive the expression for the second virial coefficient B_2 in terms of the intermolecular potential $\phi(r)$ using the approximation $\beta\varepsilon_0 << 1$. Use the Lennard-Jones potential to outline its calculation. (10)

b) Using the Mayer f-function, show that the second virial coefficient B_2 for a gas with a hard-sphere potential, $\phi(r)$, is given as $B_2 = \frac{2}{3}\pi\sigma^3$, where σ is the diameter of the sphere. Here, $\phi(r) = \infty$ for $r < \sigma$ and $\phi(r) = 0$ for $r > \sigma$. (10)

c) State Ising model and obtain the partition function (ln *Z*) for 1D in the absence of an external magnetic field up to second order perturbation. Explain the physical significance of each term in the Hamiltonian and discuss the role of the coupling constant *J*. (10)

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