MPH-004

ASSIGNMENT BOOKLET

M.Sc. (Physics) Programme
(MSCPH)

QUANTUM MECHANICS-I

Valid from 1st January, 2025 to 31st December, 2025

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2025)

Dear Student,

Please read the section on assignments in the Programme Guide for M.Sc. (Physics). A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet. The total marks for this assignment is 100, of which 40 marks are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ENROLMENT NO.: ………………………………………………

NAME: ………………………………………………

ADDRESS: ………………………………………………

COURSE CODE:…………………………..

COURSE TITLE: …………………………

ASSIGNMENT CODE: …………………..

STUDY CENTRE: ……………………….. **DATE:** ………………………………………………

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) **Submit the complete assignment answer sheets containing Part A and Part B, within the due date.**
6) The assignment answer sheets are to be submitted to your Study Centre as per the schedule. **Answer sheets received after the due date shall not be accepted. We strongly suggest that you retain a copy of your answer sheets.**
7) This assignment is **valid from 1st January 2025 to 31st December 2025**. If you have failed in this assignment or fail to submit it by December 31, 2025, then you need to get the assignment for the year 2026, and submit it as per the instructions given in the Programme Guide.
8) **You cannot fill the examination form for this course** until you have submitted this assignment. For any queries, please contact: mbnewmai@ignou.ac.in, slamba@ignou.ac.in

We wish you good luck.

Tutor Marked Assignment
QUANTUM MECHANICS-I

Course Code: MPH-004
Assignment Code: MPH-004/TMA/2025
Max. Marks: 100

Note: Attempt all questions. The marks for each question are indicated against it.

PART A

1. a) Estimate the kinetic energy of the neutrons in a neutron beam that can be used to probe lattice structures with an interatomic spacing of 0.3 nm. The mass of the neutron is 1.675×10^{-27} kg. (5)

   b) Calculate the probability current density for the wavefunction: $\psi(x) = f(x)\exp[ig(x)]$. (5)

   c) The wavefunction of particle of mass m confined to move in one dimension is $\psi(x) = Nx\exp(-\alpha x)$, $0 \le x < \infty$. Calculate the normalization constant N and the expectation value of $1/x$. (10)

   d) Show that $\left[\hat{x}^2, \hat{p}_x{}^2\right] = 2i\hbar\left(\hat{x}\hat{p}_x + \hat{p}_x\hat{x}\right)$ (5)

2. a) Consider a one dimensional potential well with an infinite barrier at $x = 0$ and a finite potential V for $x \ge L$. Solve the Schrödinger equation for a particle of mass m inside the well with an energy $E < V$. Using the boundary conditions, derive the equation for its eigen energies. (10)

   b) Calculate the probability that a simple harmonic oscillator in its ground state will be found beyond the classical turning points. (5)

   c) Consider an electron in the state

   $$\psi(\vec{r}) = N\left(\psi_{100} + 2\sqrt{2}\,i\,\psi_{210} + 4\,\psi_{21-1}\right)$$

   where ψ_{nlm_l} are the Hydrogen atom eigenfunctions. Determine the normalization constant N and the expectation values of the energy, $\hat{L}^2$ and $\hat{L}_z$. (10)

PART B

3\. a) Show that a linear combination of the degenerate eigenvectors of an operator belonging to a particular eigenvalue of the operator is also an eigenvector belonging to the same eigenvalue. (4)

b) The orthonormal bases for a three-dimensional Hilbert space is described by $|\phi_1\rangle, |\phi_2\rangle, |\phi_3\rangle$. The action of an operator $\hat{O}$ in this space is given by:

$$\hat{O}|\phi_1\rangle = 2|\phi_2\rangle; \hat{O}|\phi_2\rangle = -3|\phi_3\rangle; \hat{O}|\phi_3\rangle = |\phi_1\rangle$$

Obtain the matrix representation of this operator. (6)

c) A two state system has the orthonormal energy eigenkets $|E_1\rangle$ and $|E_2\rangle$ with eigenvalues E_1 and E_2. If the initial state of the system is given by $|\psi(0)\rangle = \frac{1}{2}\left(|E_1\rangle + \sqrt{3}|E_2\rangle\right)$, determine $|\psi(t)\rangle$. (5)

d) Derive the Heisenberg equations of motion for the simple harmonic oscillator. (5)

4\. a) i) Show that the Hamiltonian for a simple harmonic oscillator can be written in terms of the raising and lowering operators as $\hat{H} = \hbar\omega\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right)$

ii) Obtain the value of $\langle n|\hat{p}^2|n\rangle$ for the eigenket $|n\rangle$ of the simple harmonic oscillator . (5+5)

b) i) Write down the angular momentum states $|j, m_j\rangle$ for $j = 2$ and the eigenvalue of $\hat{J}^2$ and $\hat{J}_z$ for each of these states.

ii) Determine $J_+|2,1\rangle$ and $J_-|2,1\rangle$. (6+4)

c) The Hamiltonian for an electron at rest in a magnetic field B_0 along the *z*-direction is $\hat{H} = \omega_0 \hat{S}_z$, where $\omega_0 = \frac{eB_0}{m}$. Given that the initial state of the system is $|\psi(0)\rangle = \sin\alpha|\uparrow\rangle + \cos\alpha|\downarrow\rangle$, determine the state $|\psi(t)\rangle$, $\langle \hat{S}_x \rangle$ and $\langle \hat{S}_z \rangle$ at a later time *t*. (4+3+3)

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