MPH-001

ASSIGNMENT BOOKLET

M.Sc. (Physics) Programme
(MSCPH)

MATHEMATICAL METHODS IN PHYSICS

Valid from 1st January, 2025 to 31st December, 2025

ignou
THE PEOPLE'S UNIVERSITY

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2025)

Dear Student,

Please read the section on assignments in the Programme Guide for M.Sc. (Physics). A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet. The total marks for this assignment is 100, of which 40 marks are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ENROLMENT NO.: ……………………………………………

NAME: ……………………………………………

ADDRESS: ……………………………………………

COURSE CODE:…………………………..

COURSE TITLE: …………………………

ASSIGNMENT CODE: …………………..

STUDY CENTRE: ……………………….. **DATE:** ……………………………………………

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) **Submit the assignment answer sheets within the due date.**
6) The assignment answer sheets are to be submitted to your Study Centre as per the schedule. **Answer sheets received after the due date shall not be accepted. We strongly suggest that you retain a copy of your answer sheets.**
7) This assignment is **valid from 1st January, 2025 to 31st December, 2025**. If you have failed in this assignment or fail to submit it by December 31, 2025, then you need to get the assignment for the year 2026, and submit it as per the instructions given in the Programme Guide.
8) **You cannot fill the examination form for this course** until you have submitted this assignment. For any queries, please contact: drsgupta@ignou.ac.in; slamba@ignou.ac.in

We wish you good luck.

Tutor Marked Assignment
MATHEMATICAL METHODS IN PHYSICS

Course Code: MPH-001
Assignment Code: MPH-001/TMA/2025
Max. Marks: 100

Note: Attempt all questions. The marks for each question are indicated against it.

PART A

1. a) The 1-D wave equation for e.m. wave propagation in free space in given by (for $\vec{E} \| \hat{y}$)

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0$$

Solve this equation if $E_y = 0$ at $x = 0$ and $x = L$. (5)

b) The Helmholtz equation in Cartesian coordinates can be written as

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) f(x,y,z) + k^2 f(x,y,z) = 0$$

Reduce it to three ODEs. (5)

c) Using the generating function for Bessel functions of the first kind and integral order, obtain the recurrence relation

$$J_{n-1}(x) + J_{n+1}(x) = 2\frac{n}{x}J_n(x) \quad (5)$$

d) Using the Rodrigue's formula of Legendre polynomials, obtain the values of $P_3(x)$ and $P_4(x)$. (5)

e) Write an expression for generating function for Hermite polynomials. Using this expression evaluate the integral

$$\int_{-\infty}^{+\infty} x e^{-x^2} H_n(x) H_m(x) dx \quad (5)$$

2. a) What are orthonormal vectors? Show that two non-null, orthogonal vectors are linearly independent. (1+4)

b) Show that the set of all 2×2 Hermitian matrices form a four dimensional real vector space. Obtain a suitable basis for this vector space. (5)

c) Obtain the eigenvalues and the orthonormal eigenvectors for the following real symmetric matrix:

$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & -\cos\theta & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad (10)$$

d) Define contravariant and covariant tensors of rank 2. Prove that $v^i = g^{ij} a_j$ transform contravariantly. (5)

PART B

3. a) Using the method of Residues, prove that

$$\int_0^{\pi} \frac{d\theta}{1+\sin^2\theta} = \frac{\pi}{\sqrt{2}}$$ (5)

b) Show that the series $\sum_{n=1}^{\infty} \frac{z^{n-1}}{2^n}$ converges for $|z| < 2$ and find its sum. (5)

c) Write the Laurent series expansion of $\frac{e^z}{(z-1)^2}$ about $z = 1$. Determine the type of singularity and the region of convergence. (5)

d) Obtain the images of the line $x = 0$ and $y = 0$ under the transformation $w = z^2$ and prove that they intersect at right angles. (5)

4. a) Obtain the Fourier cosine transform of the function

$$f(x) = e^{-\lambda x},\ 0 < x < \infty,\ \lambda > 0$$ (5)

b) Solve the initial value problem using the method of Laplace transform

$$y'' - 2y' - 3y = 0, \quad y(0) = 1,\ y'(0) = 7$$ (5)

c) A metal plate covering the first quadrant of the xy plane has its edge along y-axis insulated. The edge along x-axis is held at an initial temperature

$$T(x,0) = \begin{cases} 100(2-x) & \text{for} \quad 0 < x < 2 \\ 0 & \text{for} \quad x > 2 \end{cases}$$

Obtain the steady state temperature distribution as a function of x and y. (5)

d) Determine the inverse Laplace transform of

$$F(x) = \frac{x+1}{x^3 + x^2 - 6x}$$ (5)

5. a) Construct the multiplication table for the group of permutations of $\{1, 2, 3\}$. (3)

b) Obtain all the proper subgroups of the permutation groups $S_3 = \{e, p_1, ..., p_5\}$ and their cosets.

Given: $e = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$; $p_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$; $p_2 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$;

$p_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ $p_4 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$; $p_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ (4)

c) Show that in the $x - y$ plane, a rotation about the origin in an anti-clockwise direction by an angle ϕ will move the points (x, y) to $(\tilde{x}, \tilde{y})$ by the matrix

$$\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$ (3)

****