MPH-001

ASSIGNMENT BOOKLET

M.Sc. (Physics) Programme
(MSCPH)

MATHEMATICAL METHODS IN PHYSICS

Valid from 1st January, 2024 to 31st December, 2024

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for M.Sc. (Physics). A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet. The total marks for this assignment is 100, of which 40 marks are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ENROLMENT NO.: …..

NAME: ……………………………………………….

ADDRESS: …………………………………………

COURSE CODE:…………………………..

COURSE TITLE: …………………………

ASSIGNMENT CODE: …………………..

STUDY CENTRE: ……………………….. **DATE:** …………………………………………

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) **Submit the assignment answer sheets within the due date.**
6) The assignment answer sheets are to be submitted to your Study Centre as per the schedule. **Answer sheets received after the due date shall not be accepted. We strongly suggest that you retain a copy of your answer sheets.**
7) This assignment is **valid from 1st January 2024 to 31st December 2024**. If you have failed in this assignment or fail to submit it by December 31, 2024, then you need to get the assignment for the year 2025, and submit it as per the instructions given in the Programme Guide.
8) **You cannot fill the examination form for this course** until you have submitted this assignment. For any queries, please contact: drsgupta@ignou.ac.in; slamba@ignou.ac.in

We wish you good luck.

Tutor Marked Assignment

MATHEMATICAL METHODS IN PHYSICS

Course Code: MPH-001
Assignment Code: MPH-001/TMA/2024
Max. Marks: 100

Note: Attempt all questions. The marks for each question are indicated against it.

PART A

1. a) Using separation of variables, solve:

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$$

where $u(x,0) = 3e^{-3x}$. (5)

b) Consider a rod of length L, whose ends are kept at a constant temperature and its lateral surface is insulated. The heat flow is described by the 1-D heat diffusion equation subject to the conditions:

$$f(0,t) = f(L,t) = 0 \text{ for } t > 0$$

$$f(x,0) = \sin\frac{3\pi x}{L} \text{ for } 0 < x < L$$

Obtain a unique solution. (5)

c) Using recurrence relation, show that

$$J_2'(x) = \left(1 - \frac{4}{x^2}\right)J_1(x) + \frac{2}{x}J_0$$

where $J_n(x)$ is the Bessel function of the first kind. (5)

d) Using the recurrence relation, show that

$$\int_{-1}^{+1} x^2 P_{n+1}(x)\, P_{n-1}(x)\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$$ (5)

e) Using the generating function, establish the relation between $H_n(x)$ and $H_n(-x)$. (5)

2. a) Obtain the eigenvalues and the corresponding eigenvectors of the matrix:

$$\begin{bmatrix} -2 & 5 & 4 \\ 5 & 7 & 5 \\ 4 & 5 & -2 \end{bmatrix}$$ (10)

b) Diagonalized the matrix:

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & -1 \\ 1 & 3 & -2 \end{bmatrix}$$

The eigenvalues of A are: $\lambda_1 = 2$, $\lambda_2 = 1$ and $\lambda_3 = -1$. (5)

c) Show that g^{ij} is a contravariant tensor of rank 2. (5)

d) Show that the vectors $\vec{u}_1 = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\vec{u}_3 = \begin{bmatrix} 1 \\ 7 \\ 2 \end{bmatrix}$ are linearly independent. (5)

PART B

3. a) Using Cauchy's residue theorem, evaluate the integral

$$\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$$ (5)

b) i) Show that the series $\sum_{n=1}^{\infty} z^n (1 - z)$ converges for $|z| < 1$ and find its sum. (3)

ii) Obtain the analytic function whose real part $u(x, y) = e^x \cos y$. (2)

c) Consider a triangle P in the z-plane with vertices at i, $1 - i$, $1 + i$. Determine the triangle P_0 which mapped P under the transformation $w = 3z + 4 - 2i$. What is the relation between P and P_0 and also calculate the area magnification. (5)

d) Obtain the Taylor series expansion of $\cos^2 z$ about $z = a$. (5)

4. a) Obtain the Fourier sine transform of the function

$$f(x) = e^{-ax} \qquad a > 0,\ 0 < x < \infty$$ (5)

b) Determine the Fourier transform of the normalized Gaussian distribution

$$f(t) = \frac{1}{\sqrt{2\pi}} \frac{1}{\tau} \exp\left[-\frac{t^2}{2\tau^2} \right] - \infty < t < \infty$$ (5)

c) Obtain the inverse Laplace transform of

$$F(S) = \frac{2S - 3}{S^2 + 2S + 2}$$ (5)

d) Using the Laplace transforms, solve the following initial value problem:

$$\frac{d^2 y}{dt^2} + \omega^2 y = \cos \omega t$$

where $y = y_0; \dfrac{dy}{dt} = \nu_0$ at $t = 0$. (5)

5. a) Show that a cyclic group is abelian. (5)

b) Using appropriate figures, show that the group S_3 is not commutative. (5)

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