MPH-008

ASSIGNMENT BOOKLET

M.Sc. (Physics) Programme
(MSCPH)

QUANTUM MECHANICS-II

Valid from 1st January, 2024 to 31st December, 2024

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2024)

Dear Student,

Please read the section on assignments in the Programme Guide for M.Sc. (Physics). A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet. The total marks for this assignment is 100, of which 40 marks are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ENROLMENT NO.: ………………………………………………

NAME: ……………………………………………………

ADDRESS: ………………………………………………

COURSE CODE:…………………………..

COURSE TITLE: …………………………

ASSIGNMENT CODE: …………………..

STUDY CENTRE: ……………………….. **DATE:** …………………………………………

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
4) Your answers should be precise.
5) **Submit the complete assignment answer sheets containing Part A and Part B, within the due date.**
6) The assignment answer sheets are to be submitted to your Study Centre as per the schedule. **Answer sheets received after the due date shall not be accepted. We strongly suggest that you retain a copy of your answer sheets.**
7) This assignment is **valid from 1st January, 2024 to 31st December, 2024**. If you have failed in this assignment or fail to submit it till its validity, then you need to get the assignment for the next year and submit it as per the instructions given in the Programme Guide.
8) For any queries, please contact: mbnewmai@ignou.ac.in, slamba@ignou.ac.in

We wish you good luck.

Tutor Marked Assignment
QUANTUM MECHANICS-II

Course Code: MPH-008
Assignment Code: MPH-008/TMA/2024
Max. Marks: 100

Note: Attempt all questions. The marks for each question are indicated against it.

PART A

1. a) Write the space translation operator in quantum mechanics for an infinitesimal translation of ε_y along the *y*-direction and ε_z along the *z*-direction. Using the properties of space translation show that $\left[\hat{p}_y, \hat{p}_z\right] = 0$. (5)

   b) Show that the angular momentum operator $\hat{L}_y$ is even under a parity transformation. (5)

2. a) Two non-interacting particles are confined in a one-dimensional box of length *a*. Calculate the eigen energy and eigen functions of the ground state and first excited state if the two particles are: i)Identical Bosons ii) Identical Fermions. (5+5)

   b) For a system of two spin half particles $\left(j_1 = j_2 = \frac{1}{2}\right)$:

   (i) list the total angular momentum eigenstates

   (ii) write down the direct product kets $\left|j_1, m_{j_1}\right\rangle \otimes \left|j_2, m_{j_2}\right\rangle$

   (iii) derive the total angular momentum states in terms of the direct product kets. (10)

3. A particle of mass *m* moves in a one-dimensional potential defined by the function:

$$V(x) = \begin{cases} U & 0 \le x < \dfrac{L}{2} \\ \dfrac{U}{2} & \dfrac{L}{2} \le x \le L \end{cases}$$

   Calculate the ground-state energy to first order in perturbation theory. (10)

4. Determine the upper bound to the ground state energy for the potential function: $V(x) = \lambda x^4$ using the trial wave function: $\psi(x) = N \exp\left(-ax^2\right)$. (10)

PART B

5. Consider the motion of a quantum particle in a potential $V(x) = \alpha x^4$. Use the WKB approximation to determine how the energy of the bound state E_n varies with n and α.
(10)

6. Consider a system described by a Hamiltonian $\hat{H} = \hat{H}_0 + \hat{V}(t)$, in which the unperturbed Hamiltonian $\hat{H}_0$ has only two eigenkets, $|1\rangle$ and $|2\rangle$, with eigen energies are E_1 and E_2, respectively. Given that $\hat{V}(t) = V_0 e^{i\omega t}|1\rangle\langle 2| + V_0 e^{-i\omega t}|2\rangle\langle 1|$, use the initial conditions $a_1(0) = 1; a_2(0) = 0$ to calculate the probability for the system to be in the state $|2\rangle$ at time *t*.
(10)

7. At time $t = 0$, a particle of mass *m* is in the ground state of the Hamiltonian $H(t) = \dfrac{p_x{}^2}{2m} + \dfrac{1}{2} m\omega^2 x^2 + kx \sin\dfrac{\omega t}{2}$. Calculate the probability that at time $t = \dfrac{2\pi}{\omega}$ the particle is in the first excited state of $H(t=0)$. (10)

8. Derive the equation of continuity starting from the Klein –Gordon equation. (10)

9. For elastic scattering by a potential $V(\vec{r})$ which has a finite range *a*, the asymptotic form for the wavefunction is:

$$\psi(\vec{r}) = A\left[e^{ikz} + f_k(\theta,\phi)\frac{e^{ikr}}{r}\right], \quad r >> a$$

Derive the expression for the differential scattering cross-section. (10)

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