

**MMTE-001**

**ASSIGNMENT BOOKLET**

**Graph Theory  
(MMTE-001)**



**School of Sciences  
Indira Gandhi National Open University  
Maidan Garhi  
New Delhi-110068  
(2022)**

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

### Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

- 1) On top of the first page of your answer sheet, please write the details exactly in the following format:

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ROLL NO.: .....

NAME : .....

ADDRESS : .....

.....

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COURSE CODE: .....

COURSE TITLE : .....

ASSIGNMENT NO.: .....

STUDY CENTRE: ..... DATE: .....

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**PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.**

- 2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.
- 3) Leave 4 cm margin on the left, top and bottom of your answer sheet.
- 4) Your answers should be precise.
- 5) While solving problems, clearly indicate which part of which question is being solved.
- 6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

We strongly suggest that you retain a copy of your answer sheets.

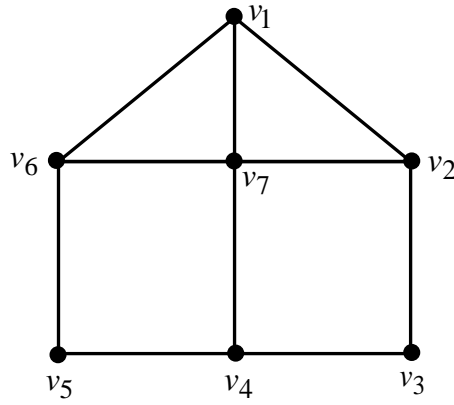
- 7) This assignment is valid only upto December, 2022. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by December, 2022, then you need to get the assignment for the year 2023 and submit it as per the instructions given in the programme guide.
- 8) **You cannot fill the Exam Form for this course till you have submitted this assignment. So solve it and submit it to your study centre at the earliest.**

We wish you good luck.

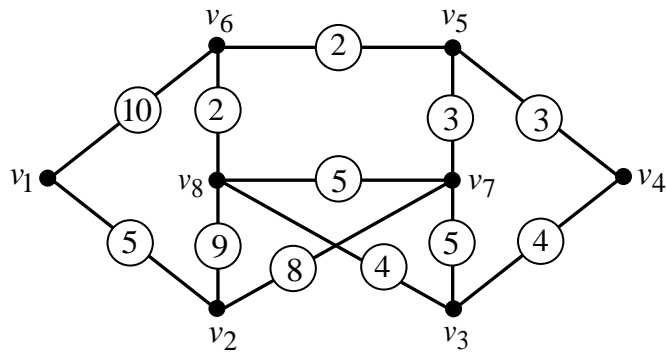
**Assignment  
(MMTE-001)**

**Course Code: MMTE-001  
Assignment Code: MMTE-001/TMA/2022  
Maximum Marks: 100**

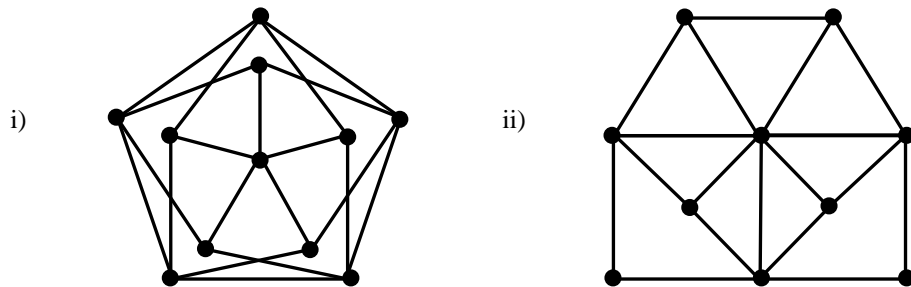
1. State whether the following statements are true or false. Justify your answers with a short proof or a counterexample. (20)
  - i) A forest has infinite girth.
  - ii) A graph with exactly two odd vertices is connected.
  - iii) The diameter of a wheel graph is 3.
  - iv) Every self-complementary graph is connected.
  - v) If  $G$  is a Hamiltonian graph, then  $G$  is Eulerian as well.
  - vi) The Petersen graph has no 3-critical subgraph.
  - vii) The thickness of a triangle-free 8-vertex graph with 25 edges is at most 2.
  - viii)  $\alpha'(K_{n,n+2}) = n$  for all  $n \geq 1$ .
  - ix) Every block of a graph is 2-edge-connected.
  - x) There exists a graph with maximum degree at most 3 and having connectivity smaller than its edge-connectivity.
  
2.
  - a) If  $P$  and  $P'$  are two edge-disjoint paths having at least two common vertices then show that  $P \cup P'$  contains a cycle. (2)
  - b) Give an example for each of the following, or justify if such a graph does not exist.
    - i) a triangle free graph having at least 9 edges and at most 6 vertices.
    - ii) a 10-vertex graph having 7 edges and exactly 2 components.
    - iii) a graph with a cut-vertex but without cut-edges
    - iv) a graph having three vertices  $u, v, w$  such that  $d(u, v) = 2, d(u, w) = 4,$  and  $d(v, w) = 3$ .
  - c) Let  $G$  be a graph with vertex set  $V(G) = \{2,5,7,10,12,14,15,18,20,21\}$  and the edge set consisting of all the pairs of vertices which are coprime. Answer the following.
    - i) Write the adjacency matrix of  $G$ .
    - ii) What are  $\delta(G)$  and  $\Delta(G)$ ? Is  $G$  regular?
    - iii) Use some software to compute the powers of  $A(G)$ . Hence, use Theorem 5 of Unit 2 to check whether  $G$  is connected or not. (7)
  
3.
  - a) Find the number of spanning trees of the following graph. (4)



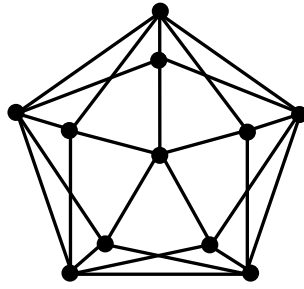
- b) Calculate the number of vertices in a complete binary tree of height 5. (2)
  - c) Use the BFS Algorithm to find a spanning tree of the graph given in part (a). (4)
4. a) Find a minimum-weight spanning tree of the following graph, using Kruskal's Algorithm. (4)



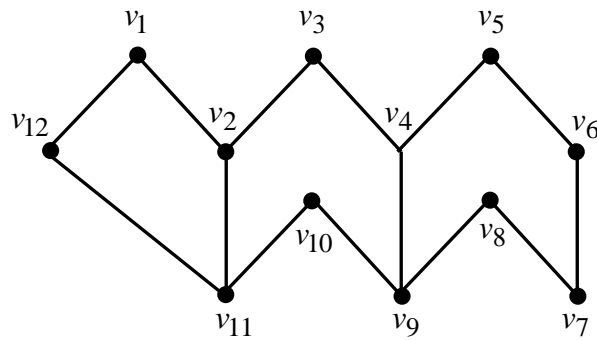
- b) Find the distance of each of the vertices from  $v_1$  in the graph given in part (a), using Dijkstra's Algorithm. (4)
  - c) Is the Petersen graph Eulerian? Is its line graph Eulerian? Justify your answers. (2)
5. a) Check whether the following graphs are Hamiltonian or not. (5)



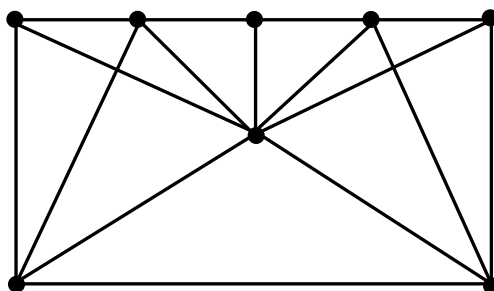
- b) Find the edge-chromatic numbers of the graphs given in part (a). (4)
- c) Check whether the following graph is planar or not. (3)



- d) Check whether the graph given in post (a)(ii) is self-dual or not. (3)
6. a) Prove that the ratio of the vertex-covering number to the independence number of a graph never exceeds the maximum degree of the graph. (3)
- b) Show that the graph given below is bipartite. Hence, verify the König-Egarvary Theorem for it. (5)



- c) Find a maximum-weight matching in a weighted copy of  $K_{5,5}$  whose weights are given by the following matrix. (5)
- $$\begin{array}{c}
 y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \\
 \begin{array}{l}
 x_1 \\
 x_2 \\
 x_3 \\
 x_4 \\
 x_5
 \end{array}
 \begin{bmatrix}
 2 & 4 & 3 & 0 & 6 \\
 8 & 4 & 5 & 1 & 3 \\
 2 & 3 & 4 & 5 & 6 \\
 1 & 3 & 2 & 4 & 6 \\
 7 & 2 & 8 & 4 & 5
 \end{bmatrix}
 \end{array}$$
- d) Given an example of a graph  $G$  with  $\kappa(G) = \kappa'(G) = \delta(G)$ . Justify you choice. (2)
7. a) Check whether the graph given below is 3-connected or not. (4)



- b) Is every 3-connected graph 3-edge-connected? Is every 3-edge-connected graph 3-connected? Justify your answers. (4)
- c) Find a maximum flow in the following network. (7)

