MMT-003

ASSIGNMENT BOOKLET

M.Sc. (Mathematics with Applications in Computer Science)

ALGEBRA

(Valid from 1st July, 2020 to 30th June, 2021)

It is compulsory to submit the assignment before filling in the exam form.

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THE PEOPLE'S
UNIVERSITY

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2020-21)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formating Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.:……………………………………………

NAME :……………………………………………

ADDRESS :……………………………………………

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COURSE CODE: …………………………..

COURSE TITLE : ………………………….

ASSIGNMENT NO.: …………………….……

STUDY CENTRE: …………………..….. DATE: ……………………..……………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Programme Centre as per the schedule made by the programme centre. Answer sheets received after the due date shall not be accepted.

We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto June, 2021. For submission schedule please read the section on assignments in the programme guide. If you have failed in this assignment or fail to submit it by June, 2021, then you need to get the assignment for 2021-22, and submit it as per the instructions given in the programme guide.

8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment
(To be done **after** studying Blocks 1-4.)

1. Which of the following statements are true? Give reasons for your answers. Marks will only be given for valid justification of your answers.

   i) In a non-abelian group of order 27, the identity conjugacy class is the only class with a single element.

   ii) A finite field with 16 elements has a subfield with 8 elements.

   iii) The number of distinct abelian groups of order $p_1^{n_1} p_2^{n_2} \ldots p_k^{n_k}$ is $n_1 n_2 \ldots n_k$, where the p_i are distinct primes and $n_i \in \mathbb{N}$.

   iv) If G is a finite group, such that $Z(G) = G$, then $o(G)$ is a prime.

   v) If X is a G-set, G a group, and $Y \subseteq X$ is G-invariant, then $X \setminus Y$ is G-invariant.

   vi) Any group of order 202 is simple.

   vii) $SL_3(\mathbb{R}) \subseteq O(3)$.

   viii) If R is an integral domain, then R/I is an integral domain, for any ideal I of R.

   ix) Every prime element of $\mathbb{Z}[x_1, x_2, \ldots, x_n]$ is irreducible.

   x) $\left| Aut(L/K) \right| = \left| Aut(L) \right| - \left| Aut(K) \right|$. (20)

2) a) Determine all the elements in the stabiliser of $\dfrac{-1 + \sqrt{-3}}{2}$, under the action of $SL_2(\mathbb{Z})$ on the upper half plane, given by $\begin{pmatrix} a & b \\ c & d \end{pmatrix}.z = \dfrac{az + b}{cz + d}$. (8)

   b) Find two distinct elements of $S_4 / < (1\ 2\ 3) >$. (2)

3) a) Explain how many distinct necklaces of 11 red and green beads are possible? (3)

   b) Prove that a group G of order 24 must have a normal subgroup of order 4 or 8. (7)
   [**Hint:** Consider a Sylow 2-subgroup P of G, and the action of G on the cosets G/P.]

4. a) Write $A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ as a product of an element of $O(3)$ and an element of $B_3(\mathbb{R})$. (5)

   b) Give an example, with justification, of an abelian group of rank 7 and with torsion group being non-cyclic of order 8. (2)

c)   Find a free group F, and $N \triangleleft F$, such that $S_3 \simeq F/N$. (3)

d)   Give a presentation of $\mathbb{Z}^3$. (5)

e)   Show that $L = \{a^n b^n \mid n \geq 0\}$ is a context-free language. (3)

f)   Give all the possible elementary divisors of a group of order 40. (2)

5.   a)   Use the division algorithm to find the inverse of $\overline{21}$ in $\mathbb{Z}_{55}$ in the form $\overline{a}, 1 \leq a \leq 55$. (5)

b)   Calculate $\left(\dfrac{29}{57}\right)$, justifying each step you use for this. (5)

c)   Prove that $\mathbb{C}^{n+2}\big/\mathbb{C}^n$ and $\mathbb{C}^2$ are isomorphic rings $\forall\, n \in \mathbb{N}$. (5)

d)   Check whether or not the ring $\mathbb{Z}\left[\dfrac{1+\sqrt{5}}{2}\right]$ is a Euclidean domain. (5)

6.   a)   Check whether or not $\mathbb{Q}(\sqrt{5}) \simeq \mathbb{Q}(\sqrt{7})$. (2)

b)   Check whether or not if L/K and K/F are Galois extensions, then L/F is a Galois extension. (2)

c)   Give an example, with justification, of a reducible separable polynomial over $\mathbb{F}_{125}$. (2)

d)   Find the splitting field of $x^6 - 4$, and its degree, over $\mathbb{Q}$. (4)

7.   i)   Show that $\mathbb{Q}(2^{1/4}, i)/\mathbb{Q}$ is a Galois extension.

ii)   Find the structure of its Galois group, G. (10)

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