BMTE-141

ASSIGNMENT BOOKLET

(Valid from 1st January, 2024 to 31st December, 2024)

BSC(G) Under CBCS

Linear Algebra

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THE PEOPLE'S UNIVERSITY

School of Sciences
Indira Gandhi National Open Universit
Maidan Garhi, New Delhi
(2024)

Dear Student,

Please read the section on assignments and evaluation in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 percent, as you are aware, has been assigned for continuous evaluation of this course, **which would consist of one tutor-marked assignment**. The assignment is in this booklet.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

	ROLL NO. : ....
	NAME : ....
	ADDRESS : ....
	....
	....
COURSE CODE :	....
COURSE TITLE :	
STUDY CENTRE :	**DATE** ....

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) **Solve the assignment on your own. Don't copy from your fellow students or from the internet. If you are found guilty of copying, your assignment will be disqualified and you will have to submit the assignment for the next session.**

7) This assignment is to be submitted to the Programme Centre as per the schedule made by the Programme Centre. Answer sheets received after the due date shall not be accepted.

8) This assignment is valid only up to December, 2024. If you fail in this assignment or fail to submit it by December, 2024, then you need to get the fresh assignment for the year 2024 and submit it as per the instructions given in the Programme Guide. **However, if you want to appear for the June 2024 (resp. December 2024) term end examination, you have submit before deadline given in the IGNOU webpage for filling the examination form for the June 2024 (resp. December 2024) examination.**

9) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So, solve it and **submit it to your study centre at the earliest.**

10) We **strongly** suggest that you retain a copy of your answer sheets.

We wish you good luck.

Assignment

Course Code: BMTE-141
Assignment Code: MTE-02/TMA/2024
Maximum Marks: 100

PART - A (30 Marks)

1) i) Find the angle between the vectors $\sqrt{2}\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{i} + \sqrt{2}\mathbf{j} + \sqrt{2}\mathbf{k}$. (2)

ii) Find the vector equation of the plane determined by the points $(1, 0, -1)$, $(0, 1, 1)$ and $(-1, 1, 0)$. (2)

iii) Check whether $W = \left\{(x, y, z) \in \mathbb{R}^3 \,|x + y - z = 0\right\}$ is a subspace of $\mathbb{R}^3$. (2)

iv) Check whether the set of vectors $\left\{1 + x, x + x^2, 1 + x^3\right\}$ is a linearly independent set of vectors in $\mathbf{P}_3$, the vector space of polynomials of degree ≤ 3. (2)

v) Check whether $T\colon\ \mathbb{R}^2 \to \mathbb{R}^2$, defined by $T(x, y) = (-y, x)$ is a linear transformation. (2)

vi) If $\{v_1, v_2\}$ is an ordered basis of $\mathbb{R}^2$ and $\{f_1(v), f_2(v)\}$ is the corresponding dual basis find $f_1\left(2v_1 + v_2\right)$ and $f_2\left(v_1 - 2v_2\right)$. (2)

vii) Find the kernel of the linear transformation $T\colon\ \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2x + 3y, 2x - 3y)$. (2)

viii) Describe the linear transformation $T\colon\ \mathbb{R}^2 \to \mathbb{R}^2$ such that

$$[T]_B = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}$$

where B is the standard basis of $\mathbb{R}^2$. (2)

ix) Find the matrix of the linear transformation $T\colon\ \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (2y, x - y)$ with respect to the ordered basis $\{(0, -1), (-1, 0)\}$. (2)

x) Let A be a 2×3 matrix, B be a 3×4 matrix and C be a 3×2 matrix and D be a 3×4 matrix. Is $AB + C^t D$ defined? Justify your answer. (2)

xi) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$. (2)

xii) Check whether $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector for the matrix $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. What is the corresponding eigenvalue? (2)

xiii) Let $C[0, 1]$ be the inner product space of continous real valued functions on the interval $[0, 1]$ with the inner product

$$\langle f, g\rangle = \int_0^1 f(t)g(t)\,dt$$

Find the inner product of the functions $f(t) = 2t$, $g(t) = \dfrac{1}{t^2 + 5}$. (2)

xiv) Find adjoint of the linear operator $T\colon\ \mathbb{C}^2 \to \mathbb{C}^2$ defined by $T\left(z_1, z_2\right) = \left(z_2, z_1 + iz_2\right)$ with respect to the standard inner product on $\mathbb{C}^2$. (2)

xv) Find the signature of the quadratic form $x_1^2 - 2x_2^2 + 3x_3^2$ (2)

Part-B (40 Marks)

1) a) Let S be any non-empty set and let $V(S)$ be the set of all real valued functions on $\mathbb{R}$. Define addition on $V(s)$ by $(f+g)(x) = f(x) + g(x)$ and scalar multiplication by $(\alpha \cdot f)(x) = \alpha f(x)$. Check that $(V(S), +, \cdot)$ is a vector space. (7)

b) Check that $B = \{1, 2x+1, (x-1)^2\}$ is a basis for $\mathbf{P}_2$, the vector space of polynomials with real coefficients of degree ≤ 2. (3)

2) a) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator and suppose the matrix of the operator with respect to the ordered basis

$$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

is $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$. Find the matrix of the linear transformation with respect to the basis

$$B' = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$ (6)

b) Show that $W = \{(x, 4x, 3x) \in \mathbb{R}^2 | x \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$. Also find a basis for subspace U of $\mathbb{R}^3$ which satisfies $W \oplus U = \mathbb{R}^3$. (4)

3) a) Find the eigenvalues and eigenvectors of the matrix $B = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 3 & 0 \\ 1 & -1 & 1 \end{bmatrix}$. Is the matrix diagonalisable? Justify your answer. (5)

b) Find Adj(A) where $A = \begin{bmatrix} 3 & 2 & 2 \\ -1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$. Hence find A^{-1}. (5)

4) a) Solve the folowing set of simultaneous equations using Cramer's rule:

$$\begin{aligned} x + 2y + z &= 3 \\ 2x - y + 2z &= 1 \\ 3x + y + z &= 0 \end{aligned}$$ (5)

b) Find the minimal polynomial of the matrix

$$\begin{bmatrix} 2 & 1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ -2 & -2 & -1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ (5)

Part C (30 marks)

1) a) Let V be the vector space of all real valued functions that are twice differentiable in $\mathbb{R}$ and

$$S = \{\cos x, \sin x, x\cos x, x\sin x\}.$$

Check that S is a linearly independent set over $\mathbb{R}$. (**Hint:** Consider the equation

$$a_0 \cos x + a_1 \sin x + a_2 x \cos x + a_3 x \sin x.$$

(Put $x = 0, \pi, \frac{\pi}{2}, \frac{\pi}{4}$, etc. and find a_i.) (4)

b) Consider the linear operator $T\colon \mathbb{C}^3 \to \mathbb{C}^3$, defined by

$$T\left(z_1, z_2, z_3\right) = \left(z_1 - iz_2, iz_1 - 2z_2 + iz_3, -iz_2 + z_3\right).$$

i) Compute T^* and check whether T is self-adjoint.

ii) Check whether T is unitary. (6)

2) a) Let $\left(x_1, x_2, x_3\right)$ and $\left(y_1, y_2, y_3\right)$ represent the coordinates with respect to the bases $B_1 = \{(1,0,0),(1,1,0),(0,0,1)\}$, $B_2 = \{(1,0,0),(0,1,1),(0,0,1)\}$. If

$$Q(X) = x_1^2 - 4x_1x_2 + 2x_2x_3 + x_2^2 + x_3^2$$

find the representation of Q in terms of $\left(y_1, y_2, y_3\right)$. (3)

b) Find the orthogonal canonical reduction of the quadratic form $-x^2 + y^2 + z^2 + 4xy + 4xz$. Also, find its principal axes. (7)

3) Which of the following statements are true and which are false? Justify your answer with a short proof or a counterexample. (10)

i) If W_1 and W_2 are proper subspaces of a non-zero, finite dimensional, vector space V and $\dim(W_1) > \frac{\dim(V)}{2}$, $\dim(W_2) > \frac{\dim(V)}{2}$, the $W_1 \cap W_2 \neq \{0\}$.

ii) If V is a vector space and $S = \{v_1, v_2, \ldots, v_n\} \subset V$, $n \geq 3$, is such that $v_i \neq v_j$ if $i \neq j$, then S is a linearly independent set.

iii) If $T_1, T_2\colon V \to V$ are linear operators on a finite dimensional vector space V and $T_1 \circ T_2$ is invertible, $T_2 \circ T_1$ is also invertible.

iv) If an $n \times n$ square matrix, $n \geq 2$ is diagonalisable then it has the same minimal polynomial and characteristic polynomial.

v) If $T_1, T_2\colon V \to V$ are self adjoint operators on a finite dimensional inner product space V, then $T_1 + T_2$ is also a self adjoint operator.