BMTC-131

ASSIGNMENT BOOKLET

Bachelor's Degree Programme
(BSCG/BAG)

CALCULUS

Valid from 1st January, 2023 to 31st December, 2023

School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068
(2023)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet, and it consists of three parts, A, B and C. The maximum marks of all the parts are 100, of which 35% are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……

NAME: ……

ADDRESS: ……

……

……

COURSE CODE: …….

COURSE TITLE: …….

ASSIGNMENT NO.: ……..…

STUDY CENTRE: ……..….. **DATE:** …….……...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto **December, 2023**. If you have failed in this assignment or fail to submit it by December, 2023, then you need to get the assignment for the year 2024 and submit it as per the instructions given in the programme guide.

8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

Course Code: BMTC-131
Assignment Code: BMTC-131/TMA/2023
Maximum Marks: 100

PART – A (40 marks)

1. Which of the following statements are true, and which are false? Give a short proof or a counter-example, whichever is appropriate in support of your answer. (10)

   i) A cubic equation with real coefficients has at least one real root.

   ii) If A and B are two sets, then:

   $$A \cup B = B \cap (A / B).$$

   iii) The greatest integer function is continuous on **R**.

   iv) The maximum possible domain of a function f, given by:

   $$f(x) = \sqrt{\frac{1-x}{x}}$$

   is $]0, 1[$.

   v) $\lim_{x \to \infty} \left(\frac{1}{2^x} - 1 \right) = -1.$

2. a) Find $\frac{dy}{dx}$ for the following cases: (6)

   i) $y = [x + (x + \sin^2 x)^3]^4$

   ii) $x^4 + y^4 = 16.$

   b) Find $\frac{dy}{dx}$, when $y = x^x + xe^x$. (4)

3. a) Let: (6)

   $$f(x) = \frac{x^2 + x - 6}{|x - 2|}.$$

   Find:

   i) $\lim_{x \to 2^+} f(x)$

   ii) $\lim_{x \to 2^-} f(x)$

   iii) Does $\lim_{x \to 2} f(x)$ exist? Why, or why not?

   iv) Sketch the rough graph of h.

   b) Is: (4)

   $$\left[\left(\frac{2-i}{1+i} - \frac{i}{2+i} \right) 3i \right]$$

a purely imaginary number? Give reasons for your answer. Also, represent this number in an Argand plane.

4. a) Find all the roots α, β, γ of the cubic equation $x^3 - 7x - 6 = 0$. Also, find the equation whose roots are $\alpha+\beta.\ \beta+\gamma$ and $\alpha+\gamma$. (5)

b) Evaluate: $\lim\limits_{x\to 0} \dfrac{e^{4x} - 1 - 4x}{x^2}$. (2)

c) For which values of the constant C is the function f continuous on **R**, where f is defined by: (3)

$$f(x) = \begin{cases} Cx^2 + 2x, & \text{if } x < 2 \\ x^3 - Cx, & \text{if } x \geq 2 \end{cases}.$$

PART – B (40 marks)

5. Which of the following statements are true, and which are false? Give a short proof or a counter-example, whichever is appropriate in support of your answer. (10)

i) A critical point of a function is its extremum point.

ii) Curve $y(x^2+1) = 3$ has an oblique asymptote.

iii) $\dfrac{d}{dx}(\sin(x^2)) = \dfrac{d}{dx}(\sin^2 x)$.

iv) $\dfrac{d}{dx}\left(\int_1^{x^4} \sec t\ dt\right) = 4x^2 \sec(x^4)$.

v) The function f, defined by $f(x) = \dfrac{1}{1+x^2}$, is integrable on every finite sub-interval in **R**.

6. a) Expand e^{2x} in powers of $(x-1)$ upto four terms. (5)

b) Verify Rolle's theorem for the function f, defined by $f(x) = x(x-2)e^{-x}$ on the interval [0, 2]. (5)

7. a) If:

$$y = x^3 \cos x,$$

then find the n th derivative of y. (5)

b) Check whether the relation:

$$R = \{(x, y) \mid xy \text{ is the square of an integer, } x, y \in \mathbf{N}\}$$

is and equivalence relation or not. (5)

8. Trace the curve:

$$y = \frac{x}{x-1},$$

stating all the properties you use to trace if. (10)

PART – C (20 marks)

9. a) Find the perimeter of the cardioids $r = 1 + \sin\theta$. (5)

b) Using the $\in -\delta$ definition of limit, prove that:

$$\lim_{x \to 1} x^3 - 2x = -1.$$ (5)

10. a) Evaluate the following integral:

$$\int \frac{(x-2)}{x^2 - 6x + 10}\, dx.$$ (4)

b) Evaluate the following integrals:

i) $$\int_1^9 \frac{(2t^2 + t^2\sqrt{t} - 1)}{t^2}\, dt.$$

ii) $$\int_0^{3\pi/2} |\sin x|\, dx.$$ (6)