BMTC-132

ASSIGNMENT BOOKLET

Bachelor's Degree Programme

DIFFERENTIAL EQUATIONS

Valid from 1st Jan, 2022 to 31st Dec, 2022

School of Sciences
Indira Gandhi National Open University
Maidan Garhi
New Delhi-110068
(2022)

Dear Student,

Please read the section on assignments in the Programme Guide for B.Sc. that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet, and it consists of three parts, Part A, Part B, Part C, covering all the blocks of the course. The total marks of the three parts are 100, of which 35% are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully:

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: ……

NAME: ……

ADDRESS: ……

……

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COURSE CODE: …….

COURSE TITLE: …….

ASSIGNMENT NO.: ……

STUDY CENTRE: ……..….. **DATE:** ……..……...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) Solve Part A, Part B and Part C of this assignment, and **submit the complete assignment answer sheets within the due date.**

6) The assignment answer sheets are to be submitted to your Study Centre within the due date. **Answer sheets received after the due date shall not be accepted.**

   We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is **valid from 1st Jan, 2022 to 31st Dec, 2022**. If you have failed in this assignment or fail to submit it by Dec, 2022, then you need to get the assignment for the year 2021, and submit it as per the instructions given in the Programme Guide.

8) **You cannot fill the examination form for this course** until you have submitted this assignment.

We wish you good luck.

ASSIGNMENT

Course Code: BMTC-132
Assignment Code: BMTC-132/TMA/2022
Maximum Marks: 100

Part A (30 Marks)

1. State whether the following statements are true or false. Give reasons for your answers. (10)

   a) The function $f(x,y)=\cos\sqrt{x^3+y^3}$ is a homogenous function.

   b) The function $f(x,y)=\begin{cases}\dfrac{x^2y}{x^4+y^2}(x,y)\neq 0\\ 0, \quad (x,y)=0\end{cases}$ is not continuous at $(0,0)$.

   c) If $f(x,y)=\begin{cases}1 & \text{if } x=0 \text{ or } y=0\\ 0 & \text{otherwise}\end{cases}$ then $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist.

   d) The range of the function $f(x,y)=\sqrt{81-9x^2-9y^2}$ is $[0,8]$.

   e) $\lim\limits_{x\to 0}\left(\lim\limits_{y\to 0}\dfrac{y\sin x}{|y|}\right)$ does not exist.

2) a) Express the following surfaces in spherical coordinates (2)

   i) $yz=2$

   ii) $y^2+z^2-x^2=1$

   b) Find the cylindrical coordinates of the points where the Cartesian coordinates are (2)

   i) $(3,3,4)$

   ii) $\left(\sqrt{5},1,2\right)$

   c) Show that the closed sphere with centre $(1,3,5)$ and radius 8 in $\mathbf{R}^3$ is contained in the open cube

   $$P=\{(x,y,z):|x-1|<10,|y-3|<10,|z-5|<10\}.$$ (3)

   d) Check whether the limit of the function $f(x,y)=\dfrac{4x^5y}{x^{10}+3y^2}$ exists as

   $(x,y)\to(0,0)$. (3)

3) a) Let $f(x,y)=\begin{cases}\dfrac{y^3}{x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 0 & \text{otherwise}\end{cases}$.

Show that f is continuous but not differentiable at $(0,0)$. (4)

b) Examine whether the second order partial derivatives of f at $(0,0)$ exist or not if $f:\mathbf{R}^2\to\mathbf{R}$ is defined by (6)

$$f(x,y)=\begin{cases}\dfrac{x^2y}{\sqrt{x+y^2}}, & xy\neq 0\\ 0, & xy=0\end{cases}.$$

Part B (40 Marks)

4) a) Solve:

$$x^2y^2(2y\,dx+x\,dy)-(5y\,dx+7x\,dy)=0 \quad (5)$$

b) Using the method of variation of parameters, solve the equation

$$\frac{d^2y}{dx^2}+a^2y=\sec ax\,. \quad (5)$$

5) a) Solve:

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\ln x \quad (5)$$

b) Solve:

$$p^2-2xyp+4y^2=0\text{, where } p=\frac{dy}{dx} \quad (5)$$

6) a) Solve $(p+q)(px+qy)=1$, using Charpit's method. (4)

b) Solve:

$$\frac{dy}{dx}=\frac{1}{x+y+1} \quad (2)$$

c) Solve:

$$x^2y''-2xy'-4y=x^2+2\ln x \quad (4)$$

7) a) Solve the equation

$$(7y - 3x + 3)\,dy + (3y - 7x + 7)\,dx = 0\,.$$ (3)

b) Using the method of undetermined coefficients, solve the equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 4x^2$$ (3)

c) Using Charpit's method, solve the equation

$$zp^2 - y^2 p + y^2 q = 0\,.$$ (4)

Part C (30 Marks)

8) a) Find the general solution of the equation

$$(x - y)\,y^2\,u_x - (x - y)\,x^2\,u_y - (x^2 + y^2)\,u = 0\,.$$ (3)

b) Find the integral surface of the PDE

$$x^2p + y^2q + z^2 = 0$$

which passes through the hyperbola

$$xy = x + y, z = 1\,.$$ (4)

c) Using Charpit's method, solve the equation

$$zp^2 - y^2p + y^2q = 0\,.$$ (3)

9) a) Find the integral curves of the differential equation: (5)

$$(D^3 - D'^3)\,z = x^3y^3$$

b) Find the compute integral of the differential equation: (5)

$$xp + 3yq = 2(z - x^2q^2)$$

10) a) Find the integral surface of the equation:

$$(x^2 - yz)\,p + (y^2 - zx)\,q = z^2 - xy$$

passing through the line $x = 1, y = 0$. (5)

b) Using Charpit's method, solve: (5)

$$p^2 + q^2 - 2px - 2qy + 1 = 0$$