ASSIGNMENT BOOKLET

**Bachelor's Degree Programme
(BSCG/BAG)**

CALCULUS

Valid from 1st January, 2022 to 31St December, 2022

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THE PEOPLE'S
UNIVERSITY**

**School of Sciences
Indira Gandhi National Open University
Maidan Garhi, New Delhi-110068**

(2022)

Dear Student,

Please read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation, **which would consist of one tutor-marked assignment** for this course. The assignment is in this booklet, and it consists of three parts, A, B and C. The maximum marks of all the parts are 100, of which 35% are needed to pass it.

Instructions for Formatting Your Assignments

Before attempting the assignment please read the following instructions carefully.

1) On top of the first page of your answer sheet, please write the details exactly in the following format:

ROLL NO.: …………………………………………

NAME: …………………………………………

ADDRESS: …………………………………………

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COURSE CODE: …………………………….

COURSE TITLE: …………………………….

ASSIGNMENT NO.: …………………………….…

STUDY CENTRE: …………………………....   **DATE:** ………………………….………………...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate which part of which question is being solved.

6) This assignment is to be submitted to the Study Centre as per the schedule made by the study centre. **Answer sheets received after the due date shall not be accepted.**

    We strongly suggest that you retain a copy of your answer sheets.

7) This assignment is valid only upto December**, 2022**. If you have failed in this assignment or fail to submit it by Dec., 2022, then you need to get the assignment for the year 2023 and submit it as per the instructions given in the programme guide.

8) **You cannot fill the Exam Form for this course** till you have submitted this assignment. So solve it and **submit it to your study centre at the earliest.**

We wish you good luck.

Assignment

PART – A (35 marks)
(Based on Blocks 1 and 2 of the course)

1. Which of the following statements are true, and which are false? Give reasons for your answers in the form of a short proof or a counterexample. (10)

    i)   The function $f: \mathbf{R} \to \mathbf{R}$ defined as $f(x) = (x - 7)(x^3 + 11)$ is an odd function.

    ii) All the cube roots of i in $\mathbf{C}$ are $z_1 = cos\frac{\pi}{2} + i \sin\frac{\pi}{2}$, $z_2 = cos\frac{\pi}{6} + i \sin\frac{\pi}{6}$ and $z_3 = cos\frac{5\pi}{6} + i \sin\frac{5\pi}{6}$.

    iii) If $f(x) = 2|x - 1|$ and $g(x) = 3x - 10$, then $fog(1) = 12$.

    iv) The domain of the function f defined by $f(x) = \sqrt{\frac{(3-x)}{(x-2)}}$ is $\mathbf{R}$-$\{2\}$.

    v) $\left(\sqrt{2}, 1, \frac{1}{2}\right) \in \mathbf{Q} \times \mathbf{Z} \times \mathbf{R}$.

2. a) If a, b and c are the roots of the equation $x^3-6x^2+10x-6=0$, find the values of $a^2+b^2+c^2$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$. (4)

    b) Using the $\varepsilon-\delta$ definition, show that $\lim_{x \to 2}(3x - 5) = 1$. (4)

    c) Find the square root of 1+i. (2)

3. a) Find the domain and range of the function f defined by $f(x) = \frac{1}{1-sinx}$. (3)

    b) Let $f(x) = \begin{cases} 1 + 2x, x \leq 0 \\ 3x - 2, 0 < x \leq 1 \\ 2x^2 - 1, x > 1 \end{cases}$

        i) Check whether f is discontinuous. If yes, find where?

        ii) Give a rough sketch of the graph of f. (5)

    c) Express the function g, defined by $g(x) = \frac{1}{\sqrt{x + \sqrt{x}}}$, as a composition of three functions. (2)

4. Find the following limits: (5)

    (i) $\lim_{x \to 2} \frac{x^2+4x-1}{x^2-2x}$

    (ii) $\lim_{x \to 0} x^2 \cos\frac{1}{x}$

PART – B (40 Marks)
(Based on Blocks 3 and 4 of the course)

5. a) If $f(x) = \sin^{-1} x$, show that $(1 - x^2)f''(x) - xf'(x) = 0$. Hence, prove that $f^{(n+2)}(0) = n^2 f^n(0)$. (4)

    b) Find the maximum height of the curve $y = 4\sin x - 3\cos x$ above the x-axis. (3)

    c) Verify Rolle's theorem for f on [-1, 1] defined by $(x) = x^4 - 4x^2 + 7$. (3)

6. a) Find the approximate value of $\sqrt[3]{0.99}$ upto 4 places of decimal. (4)

    b) Differentiate y w.r.t. x in the following cases: (6)

    i) $y = \sqrt{\sin(\sqrt{x})}$

    ii) $y = e^{\sin x} + x\ln x$

7. Which of the following statements are true and which are false? Give reasons for your answers, in the form of a short proof or a counter example.

    i) The function f defined by f(x) = tan(2x) is a periodic function with period π.

    ii) The function $f : R \to R$, defined by $f(x) = 1 - |x|$ is differentiable at x=1.

    iii) The function $f : [3,4] \to R$ defined by $f(x) = x^2 - x$ is monotonic in its domain.

    iv) Every continuous function is differentiable.

    v) Every curve over **R** has a point of inflection. (10)

8. Trace the curve $y = \frac{8}{4-x^2}$, and state all the properties you use to trace it. (10)

PART – C (25 Marks)
(Based on Block 5 of the course)

9. Integrate the following functions w.r.t. x: (9)

    i) $\int \frac{x-1}{x^3-x^2-2x} \, dx$

    ii) $\int_2^4 x\ln x \, dx$

    iii) $\int \frac{(\cot^{-1} x)^4}{1+x^2} \, dx$

10. (a) If $I_n = \int_0^\infty e^{-x} \sin^n(x) dx$, prove that $(1 + n^2)I_n = n(n - 1)I_{n-2}$ for $n \geq 2$. (5)

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b) Find the length of the curve given by $x = t^3, y = 2t^2$ in $0 \le t \le 1$. What is the slope of the curve at t = ½. (5)

11. Which of the following statements are true? Give reasons for your answers, in the form of a short proof or a counter example. (6)

i) $\frac{d}{dx}\left[\int_{x^2}^{\frac{\pi}{2}} e^{2\cos t}\, dt\right] = -2xe^{2(\cos x^2)}$

ii) $|x|$ is integrable in [-2, 2].

iii) $\int_a^b x\, dx = \frac{b^2-a^2}{2}$.